SHIP FORM, RESISTANCE AND SCREW PROPULSION

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AND
SCREW PROPULSION

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PREFACE

THIS work is intended mainly for the use of naval architects, engineers, and draftsmen. It will also be found of use to the student who has an elementary knowledge of ship resistance. The subject has been treated from a practical point of view, and theory has been introduced only where it has a direct practical bearing.

All existing experimental data on models or ships have been carefully examined and compared, and those likely to be of permanent use are reproduced. The mode of presenting resistance and power results has been a matter of some difficulty. It was necessary that these results should be in a form applicable to all sizes of ships, and for this Froude's "constant" notation has undoubtedly greater advantages than any other. Nevertheless, where greater clearness has been possible by departing from this notation, other methods have been adopted.

The sections on form and variation of form have been given in considerable detail, and the separation of experimental data in the manner adopted has rendered clear and intelligible much that was before contradictory. In the analysis and comparison of the large mass of experimental data the author cannot expect to have avoided all error, and where any such may be detected he would be glad to have it pointed out.

To a large extent the book is based upon the work of experiment tanks, and it hardly need be said that such a book finds its real

place in guiding the designer to what should be tank tested, rather than to help him to dispense with such tests. It should, however, be of great use in making preliminary estimates for power in the early stages of a design, particularly when new types or large departures from practice are contemplated.

My thanks are due to Mr. J. Kent, one of my colleagues at the National Experiment Tank, for his assistance in the preparation of the diagrams and the checking of the calculations given in the book.

G. S. B.

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SHIP FORM, RESISTANCE AND SCREW PROPULSION

PART I

SHIP FORM AND RESISTANCE

CHAPTER I

NOMENCLATURE

§ 1.—For the sake of clearness the various terms, symbols, etc., used throughout Part I. of the book are here gathered together and their meaning clearly stated. This enables the reader at any time to ascertain exactly what any term may mean, and helps to clear the text of confusing repetitions.

Parallel body is that portion of the immersed body, any transverse section of which has the same area and the same shape.

The **fore body** is the immersed body forward of the midship section, this latter being situated midway between the fore and aft perpendiculars.

The **after body** is the immersed body aft of the midship section.

The **entrance** is the immersed body forward of the parallel body, or, if the latter is *nil*, forward of the cross-section of greatest area.

Length of entrance is the length from the fore perpendicular to the section at which the entrance ends.

The **run** is the immersed body aft of the parallel body, or, if the latter is *nil*, aft of the cross-section of greatest area.

The **length of run** is the length from the after perpendicular to the after end of the parallel body.

Block coefficient is the ratio $\dfrac{\text{immersed volume}}{L \times B \times D} = \beta$.

Prismatic coefficient is the ratio $\dfrac{\text{immersed volume}}{L \times (\text{largest section area})} = P$.

Prismatic coefficient of fore body is the ratio

$$\frac{\text{immersed volume forward of the midship section}}{\frac{1}{2} L \times (\text{largest section area})}.$$

Prismatic coefficient of entrance P_e is the ratio

$$\frac{\text{immersed volume of entrance}}{L_e \times (\text{largest section area})}.$$

The prismatic coefficients of the after body and of the run are similarly defined.

Largest section coefficient is the ratio

$$\left(\frac{\text{area of largest section}}{B \times D}\right) = \frac{A_M}{B \times D}.$$

L, the length in feet taken between perpendiculars;

B, the breadth } to mean plating line at the midship

D, the mean draft } section, also in feet;

Δ, displacement of the ship in tons;

V, speed of ship in knots;

w, weight of salt water per cubic foot;

R, total resistance of the ship in tons;

R_F, resistance due to skin friction, to which Froude's law of comparison is not applicable;

R_W, the residuary resistance to which Froude's law is applicable.

§ 2. Froude's Constants.—These constants are largely used in experimental work, as they afford the best means of making a comparison of forms at a given speed for given displacement. The underlying principle is a comparatively simple one. All dimensions are expressed in terms of the unit U, which is equal

to the length of the side of the cube having contents equal to the immersed volume of the ship in cubic feet. Thus :—

The length constant

$$(M)=\frac{L}{U}=\frac{L}{(35\times\Delta)^{\frac{1}{3}}}=\cdot3057\left(\frac{L}{\Delta^{\frac{1}{3}}}\right).$$

The skin constant

$$(S)=\frac{\text{the wetted surface } S}{(35\times\Delta)^{\frac{2}{3}}}=\cdot0935\left(\frac{S}{\Delta^{\frac{2}{3}}}\right).$$

An approximate formula for (S) is

$$(S)=3\cdot4+\frac{(M)}{2},$$

which gives an indication of the variation of (S) with (M).

The speed is expressed in such a form that when a comparison of different forms is to be made on a displacement basis this term shall remain constant at "corresponding speeds."

It must therefore take the form $\frac{V}{\Delta^{\frac{1}{6}}}$. Since the speed of a wave whose length is $\frac{U}{2}$ is

$$=\sqrt{\frac{g}{2\pi}\times\tfrac{1}{2}\times(35\Delta)^{\frac{1}{3}}},$$

the ratio of the ship's speed V to the above is of the right order, and this ratio has been chosen as the means of expressing the relation between the speed and size of the ship. Hence the speed constant

$$(K)=\frac{V\times\frac{6080}{3600}}{\sqrt{\frac{g}{2\pi}(\Delta\times35)^{\frac{1}{3}}\times\frac{1}{2}}}=\cdot5834\frac{V}{\Delta^{\frac{1}{6}}}.$$

This, of course, becomes unity when V is equal to the speed of the wave whose length is $\frac{U}{2}$.

The relation between the speed and the length may similarly

be expressed by the ratio of the speed V to that appropriate to a wave of length $\left(\frac{L}{2}\right)$. Hence the length-speed constant

$$(L)=\frac{V\times\frac{6080}{3600}}{\sqrt{\frac{g}{2\pi}\left(\frac{L}{2}\right)}}=1\cdot055\frac{V}{\sqrt{L}}.$$

The resistance constant (C) is simply the Admiralty constant inverted and

$$=\left(\frac{R}{\Delta^{\frac{2}{3}}V^2}\right).$$

This remains the same for all similar forms at corresponding speeds, so that a curve of (C) to a base of (K) remains the same for all ships of the same form irrespective of their size, if the variation of the skin friction with length of surface is neglected. The unit of the constant is chosen, so that when (K) is unity,

$$(C)=\frac{R}{\Delta}.$$

For this (C) must equal

$$\frac{R}{\Delta (K)^2}=\frac{R}{\Delta\left\{\frac{V}{\Delta^{\frac{1}{6}}}\times\cdot5834\right\}^2}=\frac{RV\left(\frac{2240\times6080}{33000\times60}\right)}{\Delta^{\frac{2}{3}}V^3\times\cdot34\left(\frac{2240\times6080}{33000\times60}\right)}=\cdot4271\frac{\text{E.H.P.}}{\Delta^{\frac{2}{3}}V^3}.$$

As invariably plotted (C) is one thousand times the above, and should therefore be written :—

$$(C)=427\cdot1\left(\frac{\text{E.H.P.}}{\Delta^{\frac{2}{3}}V^3}\right).$$

Another constant to which reference is made in the text of the book is (P). This gives the ratio of the speed of the ship to the speed of a wave whose length is $(P\times L)$.

This length PL is used as a measure of the statical wave length

of the ship. The speed v corresponding to a wave of this length is given by :—

$$v=\sqrt{\frac{g(P\times L)}{2\pi}},$$

so that

$$P=\frac{V}{v}=\cdot 746\frac{V}{\sqrt{P\times L}}.$$

It will be seen that the ratio of the ship's speed to this speed v gives a fair indication of the character of the resultant system of waves the ship tends to form, and that when the ship's speed is equal to that of waves having lengths $\frac{1}{2}$, $\frac{1}{3}$, etc., of PL, the corresponding P values are $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{3}}$ etc., called P_2 P_3.

Where diagrams of C values are given, these are for fixed length of ship, generally 400 feet. For vessels of other lengths some allowance for variation of skin friction with length becomes necessary. This allowance takes the form of a small deduction for greater lengths, and a somewhat larger addition for smaller lengths. It varies to a certain extent with the fulness of the form, but not very much, and for all ordinary forms the corrections given in Table 36 may be used in making any estimate from the diagrams.

CHAPTER II

STREAM LINE MOTION

§ 3.—Stream lines play such an important part in the resistance of a ship that it is desirable for the general reader to have a fair understanding of them, and a brief outline of the theory, its principles, and some examples which will serve to illustrate the theory and show its latest development are given here.

A ship-shaped form, if towed through an infinite perfect fluid at any uniform speed, will set up motions in that fluid, and at any point fixed relative to the form the relative motion of the stream and the body will always be the same. If the form is at rest in a stream which is flowing past it with uniform velocity, then again it may be said that the path of every particle passing through any point fixed relative to the form is always the same. In other words, the *relative* motions are the same whether the body moves through the fluid or the fluid moves past the body.

The path so traversed by a particle relative to the form is called a *stream line*, and for any fully submerged body moving in a given manner in a perfect fluid there is but one set of such stream lines. By a perfect fluid is meant any fluid which is incapable of experiencing or transmitting a tangential or shearing force. Sea water, the fluid with which we are particularly concerned, does not come under this definition, it being to a slight extent viscous, *i.e.*, capable of transmitting shearing force. The effect of this imperfection will be considered later, but outside of the local disturbance due to it the pressure effects and the stream lines of any fair form may, for practical purposes, be regarded the same as if the water were a perfect fluid.

Consider a pipe bent into the form in the figure having a perfect fluid flowing through it at uniform velocity. Since the ends of

the pipe are in the same straight line, if the cross-section areas of the ends of the pipe are the same, the velocities will be the same. Since the fluid is frictionless, no energy is dissipated in friction, and the energy of the particles remains the same at every point. This energy may be represented by H, where

$$H = \frac{p}{w} + \frac{(v)^2}{2g} + z,$$

v being the velocity in feet per second,
z the altitude of the particle in feet,
p the pressure at the particle in lbs.,
w the density in lbs. per cubic foot.

It follows that with any change of velocity of the particles there is a corresponding change in pressure or altitude. These pressures

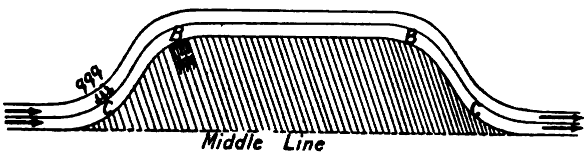


FIG. 1.

throughout the length of the pipe cancel one another, and there is no tendency of the fluid to give longitudinal motion to the pipe. But there is a definite tendency to movement at the bends B and B, and if the pipe were elastic it would be necessary to support it by forces qqq at these parts. Similar supporting forces are required at C to enable the pipe to retain its shape. These supporting forces may be supplied in any manner selected. Let it be supposed that on the inner side there is a body as shaded, and on the outer side there is another elastic tube next to this one, with the same fluid flowing through it at the same velocity. These two tubes then support each other at their common faces. By imagining large numbers of such tubes placed together so that they support each other, and that their walls become thinner and thinner, we at last get to the condition of a number of stream tubes passing a body at a uniform velocity. We see, too, that this body is subjected to pressures varying from

point to point, and becoming more intense near the ends of the moving form and less near the centre, where its cross-section area is greatest. The effect of this pressure upon the resistance will be dealt with hereafter. At present it is better to confine ourselves to the investigation of a few cases in detail, examining both the shape of the stream lines and the distribution of the pressure along the forms.

§ 4.—At present the best known method of doing this involves the use of hydrodynamic " sources " and " sinks." The mathematics is too complicated to give here, and is not necessary for a general understanding of the subject. Briefly it may be said that a source is a point in a mass of fluid at which fluid is being continually introduced. If the mass is otherwise at rest, the fluid will spread in radial lines *from* the source. If the fluid be abstracted continually at a point, this point is called a " sink," the stream lines again being radial; but now the motion is *towards* the sink. If the flow is restricted between parallel planes quite close together, the motion is in two dimensions, and the sources and sinks are called " two dimensional." On the other hand, if the flow takes place in *every* direction, then we have three dimensional sources and sinks. The strength of a source or sink is the amount of fluid introduced or abstracted at it in unit time. By combining sources and sinks with a stream of uniform velocity various patterns of stream lines can be obtained, and if the sources and sinks are placed on a line parallel to the direction of the uniform stream, and the total strength of the sources equals that of the sinks, symmetrical flow will be obtained and one of the stream lines will be a closed curve. By a proper adjustment of the strengths of the sources and the velocity of the uniform stream, this closed stream line can be given any desired shape, of which the stream lines can be plotted and the variations in pressure along the form can be obtained.

The simplest cases are those derived from a single two-dimensional source and sink combined with a parallel flow. This, however, gives blunt-ended ovals as the closed curves, which bear little likeness to the ordinary ship form. By imagining

a line of such sources and sinks of varying strength the stream lines for shapes similar to the level lines of a ship * can be obtained. This has been done, and Fig. 2 shows a typical set of lines for a sharp-ended form, together with pressure curves along the form, and along a stream line which, when undisturbed, was at a distance of one-tenth the length from the centre line.

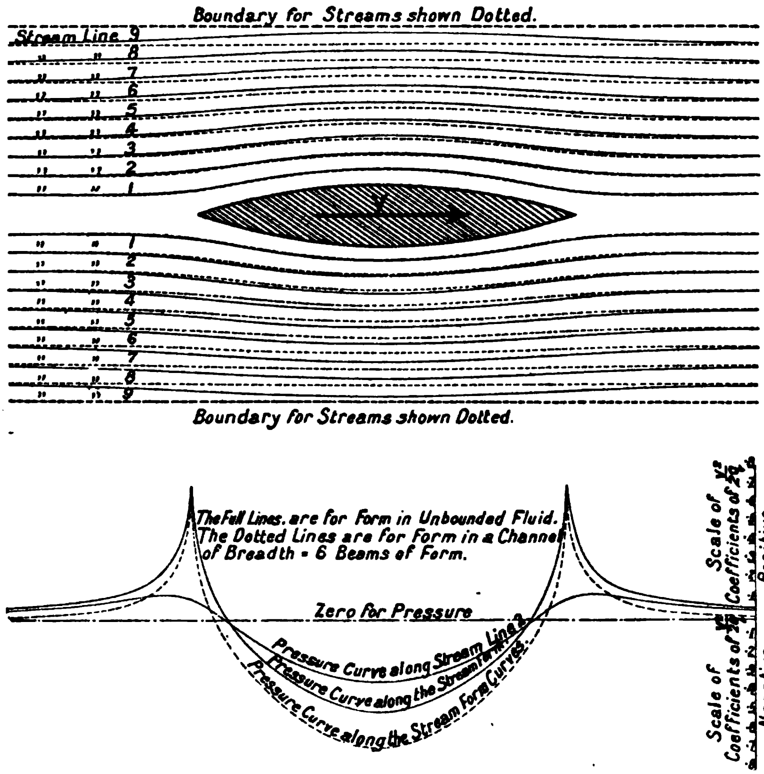


FIG. 2.—Stream Lines around Ship-shaped Form and corresponding Pressure Curves.

If instead of considering two-dimensional cases we turn to three dimensions, the problem can be treated in much the same way, but the work is now much more complicated and laborious. Isolated space sources and sinks will give the stream lines around an oval of revolution having very full ends, and a line of such

* D. W. Taylor, Trans. I. N. A., 1894—5.

sources and sinks will give the stream lines past an oval of revolution, such as a balloon or submarine, having pointed or blunt ends, the degree of sharpness and the shape of the solid depending on the strengths of the sources and sinks at various points along the central line. Vessels of such form are, however, almost non-existent, and in any case the results have no application unless the form is either fully submerged or has its axis of revolution in the water plane. Such a case, therefore, is of little general interest.

For most practical purposes the actual paths of individual stream lines is of little importance, the main thing being the total effect of the changes in all the stream lines upon the ship as a whole. For this it is required to know only the pressure disturbance against the form considered. This problem, for a perfect fluid, can be solved by making an assumption which experience seems to warrant as accurate. The stream lines on a number of ship-shaped models, obtained experimentally by Mr. Taylor, show that at all moderate speeds for the ship the motion is seldom or never in a horizontal plane, *i.e.*, is not along the level lines generally. Several sets of experiments made at the William Froude tank with a high-speed model and the photographic work of Ahlborn show much the same thing. It follows that the consideration of the streams around a level line will not lead to an accurate indication of what is going on around the ship. Moreover, there is ample experimental evidence that the chief wave-making of a ship is far more dependent in its characteristics upon the curve of areas than upon the shape of individual level lines, and this is particularly true with the phase of the question considered here, viz., the distribution of stream line pressure around the ship. For these reasons it seems probable that a better indication of what takes place will be obtained by the use of a stream form similar in shape to the *curve of areas* for the ship, and not a level line. This course has the advantage not only of being independent of the ship's actual lines, but, since it only involves streams in two dimensions, it is not nearly so laborious as even the simplest three-dimensional case.

§ 5.—Fig. 3 shows the pressure variations along forms whose shapes are given in Fig. 4. The characteristics of these forms are as follows :—

GENERAL DIMENSIONS OF STREAM FORMS.

Form.	A.	B.	C.	D.	E.
Length units .	188	192	188	192	192
Midship section in area units .	25·6	25·6	25·6	25·6	25·6
Prismatic coefficient	·50	·55	·65	·75	·76

It will be seen that these have the common characteristic of all such forms—pressure humps near the ends and decreased pressure near the middle. It will also be seen—

(*a*) that the humps are generally more emphatic the fuller the form, and occur nearer the ends of the form up to a certain prismatic coefficient, beyond which increase in fulness has little effect ;

(*b*) that the forms with the larger prismatic coefficients have a curious depression in the pressure curve occurring between the maximum pressure and the mid-length of the body.

If the stream forms are given an angular termination the pressure hump has its maximum ordinate at the end of the vessel, and the larger the angle of entrance, the more peaked the curve becomes at this part, the larger becomes the ordinate of the pressure curve in front of the form, and the more sudden is the drop from the positive to the negative pressure—*i.e.*, this negative pressure extends over a greater length of the form the greater the angle of entrance becomes. These changes are much as would be expected from a study of the previous work, since increase in angle of entrance is usually accompanied by extra fulness. Such

a stream form and its pressure curve are shown in Fig. 2, the ratio of width to length being 1 to 6 and the entrance angle 34 degrees.

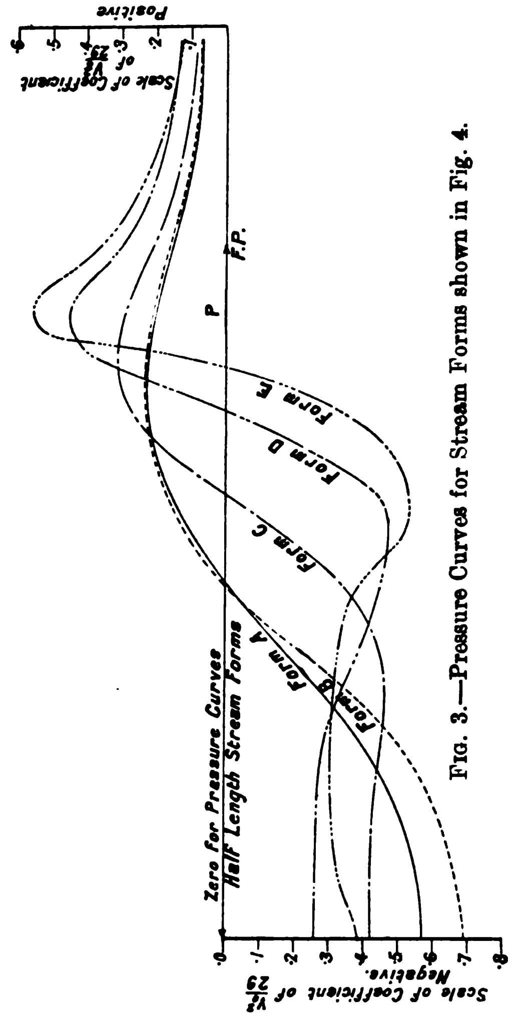


FIG. 3.—Pressure Curves for Stream Forms shown in Fig. 4.

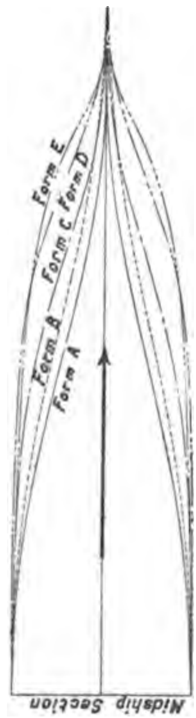


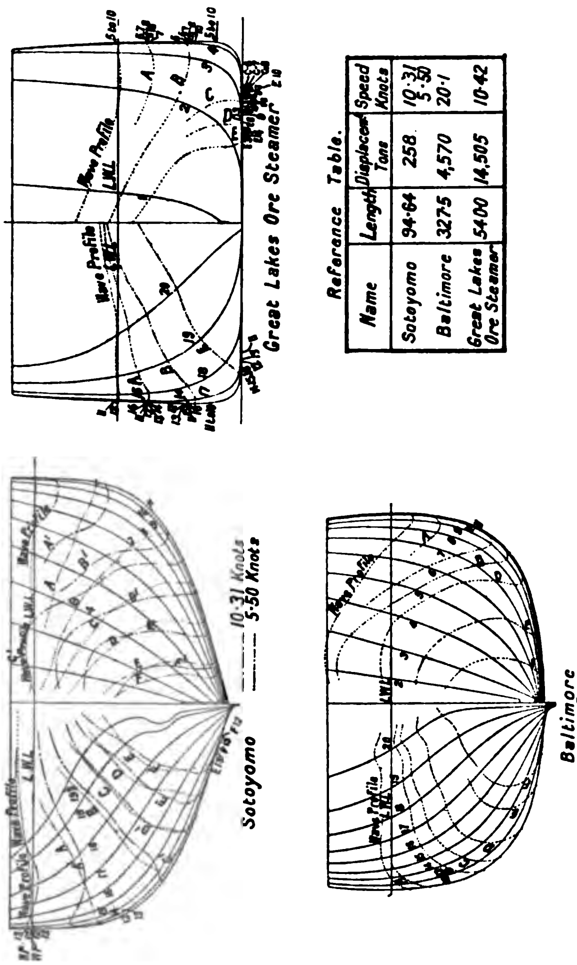
FIG. 4.—Stream Forms of varying fulness.
The forms are symmetrical about the midship section.

§ 6. Limited Fluid.—So far the motion considered has been that in a limitless fluid. As the confines of a body of water become more restricted, so the boundaries begin to affect the motion of the water. If in the two-dimensional case the form is moving in

a channel having straight boundaries the equations for the separate simple source and sink with these boundaries can be obtained, and by combining a source and sink the blunt-ended oval and its stream lines under these new conditions are found. Just as in the case of the limitless fluid, by imagining a line of such sources and sinks of varying power, the stream lines for the ship-shaped form with the fixed boundaries can be obtained. The work is intricate and difficult, and the reader is not advised to attempt it unless he has considerable spare time. In order to show this effect the pressure curve for the stream form, for which results in a limitless fluid are given in Fig. 2, has been obtained with boundary walls whose distance apart is the length of the form. This curve is shown dotted in the figure. It will be seen that the presence of these walls has caused an increase in the pressure in the locality of the moving form amounting approximately to 30 per cent. This case may be considered as similar in effect to that of a vertical-sided ship travelling along a narrow channel, the motion of the water being practically all in level planes. Considering only that part of the diagram below the centre line, it represents a ship having an infinite beam travelling in shallow water of depth equal to half the ship's length. This is by far the more practical case of the two, and, although not rigidly accurate, the pressure curves in the figure must indicate the change which takes place when the form runs from deep into shallow water.

§ 7. **Experimental Results.**—In passing from these stream forms to the actual ship two new factors are introduced—first, that the ship travels at the surface of the water and is therefore accompanied by waves, and, second, the ship's shape changes gradually from a line at stem and stern to an ellipse or rectangle at amidships. The disturbance of the surface streams by the waves will be naturally felt for some considerable depth. Since these waves are created in the first place by the travelling pressure disturbance due to the stream line motion, there must be a certain amount of give and take between the stream line and the wave pressures, which will modify to a certain extent the general stream line

changes. With this reservation, which is of importance only at speeds which are high for the length, it may be considered that the diagrams already given do represent the general phenomena. At each end there is a widening out of the streams, with the



Reference Table.

Name	Length	Displacem^t Tons	Speed Knots
Sotoyomo	94·64	258	10·31 / 5·50
Baltimore	327·5	4,570	20·1
Great Lakes Ore Steamer	540·0	14,505	10·42

Fig. 5.—Stream Lines around Ship Models.

attendant excess of pressure, and at the shoulders and amidships there is the thinning down of the streams and corresponding decrease of pressure.

Experimenters have in recent years attacked this problem.

The stream lines of various fish-shaped forms have been obtained by Professor Hussy and those for some typical ship-shaped forms by Taylor. Fig. 5 shows three sets of the latter—one for a single screw boat of rounded bilge, the second for a form having a moderately round midship section, the third for a full form having a large amount of parallel body. It will be seen that at the surface and along the ship's vertical sides the streams follow the wave profile generally. But those streams which pass beneath the turn of the bilge travel more nearly along the buttocks in the fore body, show a marked outward movement along the flat floors, and turn in to a diagonal line at the stern. The spreading of the streams under the flat floors must be due to either a flattening out or a decrease in velocity of the streams due to the frictional action of the surface. At the bilge turn along the amidship portion of the ship there is a comparatively marked widening out of the side and bottom stream lines on all the forms tried. This seems to suggest that along this portion of the ship the flow is more or less critical and inclined to eddy-making, and it is only when the bilge is well rounded that this feature disappears.

§ 8. The Law of Comparison.—This law, which is due to W. Froude, states that :—

The resistances of similar ships are in the ratio of the cubes of their linear dimensions, when their speeds are in the ratio of the square root of their dimensions.

The speeds which are connected by this relation are known as *corresponding speeds.*

This law applies only to that resistance for which the dynamic conditions are similar, irrespective of size. It is known that this is not the case so far as the frictional resistance of a ship is concerned, and the law does not apply to it. For this reason the results of experiments with models need to be corrected for friction when they are applied to the ship.

The law is now generally accepted, but the following proof is given for those interested in the matter. The force which any

body exerts can be measured by the acceleration it will produce in a given mass, or if

F is the force,
m is the mass,
a is the acceleration,
t is the measure of time,
v is velocity,
l is distance,

then

$$F \propto m \times a.$$

The velocity of a body is the ratio

$$\frac{l}{t},$$

and acceleration is

$$\frac{\text{velocity}}{\text{time}},$$

or

$$\frac{l}{t^2}.$$

Since the mass varies as the cube of dimensions it can be written

$$m = w \times l^3,$$

so that force now can be written

$$F \propto wl^3 \times \frac{l}{t^2} = w\frac{l^4}{t^2} = w\frac{l^5}{lt^2},$$

remembering that velocity is

$$\frac{l}{t},$$

then

$$F \propto (w \times l^3) \times \left(\frac{v^2}{l}\right).$$

For any change in dimensions, therefore, provided that $\left(\frac{v^2}{l}\right)$ is constant, F must vary as $w \times l^3$.

For example, the energy of a wave is proportional to breadth $\times$ length $\times$ (height)^2, *i.e.*, varies as the fourth power of the dimensions.

The velocity is proportional to $(\text{length})^{\frac{1}{2}}$.

A ship which is n times the size of another will create, at corresponding speeds, waves which are n times the size of those made by the other. The energy dissipated in unit time by each ship in wave-making is proportional to—

$$\text{wave length } (\text{height})^2 \text{ breadth} \times \left(\frac{\text{velocity}}{\text{wave length}}\right).$$

The resistance is therefore proportional to—

$$(\text{height})^2 \times \text{breadth, } i.e., \ (\text{dimensions})^3,$$

or this form of resistance comes under the law.

CHAPTER III

SKIN FRICTION RESISTANCE

§ 9.—In the previous section it has been assumed that the fluid through which the body is moving is perfect, *i.e.*, frictionless. Actually one of the largest causes of resistance is the frictional character of the water. This imperfection of water affects the resistance in two ways—first, in the production of what is now known as skin friction, and, second, in aiding eddy-making. The skin resistance is due to the tangential forces between the surface of the body and the layer of water with which it is in contact producing in the water a forward motion, so that the form is accompanied by a comparatively thin layer of water, the forward velocity of which increases with the length of the surface.

The nature of this surface resistance is but little known. The frictional belt formed was found by Hele Shaw to consist of a thin film having straight line flow in contact with the body and sinuous motion outside of this film. To what extent this may be true for a ship is not known. His experiments indicate that neither air nor lubrication of any kind admitted to the surface had much effect upon the phenomena.

Professor Ahlborn has experimented with plain planks and found that increase of velocity did not increase the thickness of this belt, but only the accelerations of the particles inside it. He has also shown that the thickness of the frictional belt increases but slowly with length of surface, particularly with a very smooth plank. This belt of water is being continually renewed at the forward end and left behind at the after end in what is known as the frictional wake—*i.e.*, a following current of fairly uniform width, flanked by innumerable eddying whirls where the following current is in contact with the still water.

The amount of energy dissipated in this way has been investigated by Mr. William Froude, by Tideman, and more recently by Dr. Stanton and Herr Gebers. Mr. Froude's experiments were made with thin planks of a uniform depth of 21 inches, varying in length up to 50 feet and in speed up to 800 feet per minute. The planks were towed through the water and the resistance recorded in much the same manner as is described later for ship models. Every care was taken to eliminate air, wave, and eddy resistance, so that the results may be taken as giving pure frictional resistance. He found the resistance to be given by a formula of the form

$$R=f\times A\times V^{n}.$$

where

R is the resistance in lbs.,
A is the area in square feet,
V is the speed in feet per second.
f and *n* are constants, depending on the length and nature of the surface, and are given in Table 1, p. 20, for fresh water.

These planks were also tried with various "compositions" used for painting ships' bottoms, and it was found that their resistance was practically the same as for the paraffin and varnish surface, and it may be generally assumed that a fair form with a smooth coat of paint or composition on it will have the same frictional value as a paraffin surface. If, however, the surface when painted feels *gritty* to the hand even if the grit is held in a good coat of varnish, an increase in resistance (as indicated by the results with a fine sandy surface above) may be expected.

Tideman's results are slightly in excess of Froude's, but the differences are not important, averaging about 4 per cent. all over.

Herr Geber's were somewhat less, being generally 4½ per cent. lower, but his experiments were made under slightly different conditions, the top edge being awash, and not immersed, as in Froude's work.

TABLE 1.

Nature of Surface.	Length of Surface or Distance from Cut Water in Feet.											
	2 Feet.			8 Feet.			20 Feet.			50 Feet.		
	f.	*n.*	*k.*	*f.*	*n.*	*k.*	*f.*	*n.*	*k.*	*f.*	*n.*	*k.*
Varnish .	·0041	2·0	·0039	·0046	1·85	·00374	·0039	1·85	·00337	·0037	1·83	·00335
Paraffin .	·00425	1·95	·00414	·0036	1·94	·003	·00318	1·93	·0028			
Calico .	·010	1·93	·0083	·0075	1·92	·006	·0068	1·89	·0057	·0064	1·87	·0057
Fine sand .	·008	2·0	·0069	·0058	2·0	·0045	·0048	2·0	·00384	·0040	2·06	·0033
Medium sand	·009	2·0	·0073	·0063	2·0	·0049	·0053	2·0	·0046	·0049	2·0	·0046
Coarse sand	·011	2·0	·0088	·0071	2·0	·0052	·0059	2·0	·0049			

The columns k give the "f" values for the last square foot of a surface, whose length is equal to that specified in the heading.

In order that the results obtained with planks may be used to estimate the frictional resistance of a ship's surface, it is necessary to extend the results—

(*a*) to any length up to 800 or 900 feet;

(*b*) to speeds more than four times the highest speed of the experiments;

(*c*) from a plane surface to the surface of a solid.

Extension to Longer Lengths.—Since increase in length from 20 to 50 feet has no appreciable effect upon the value of *n*, this may be taken as the same for all lengths above 50 feet. The "*f*" value for long lengths may be obtained by assuming that the frictional coefficient of the first 50 feet is the same as that of a 50-foot plank, regardless of the ship's length, and that the remainder of the length has the same frictional coefficient as the last foot of the 50-foot plank. Mr. R. E. Froude finds that with models both the "*f*" and "*n*" values may be regarded as substantially the same for either paraffin or varnish surface, the latter being taken as 1·825, which thus enables ship results to be estimated from the model results with comparative ease. The values of "*f*" obtained in this way corrected for salt water are given in the following table:—

TABLE 2.

"*f*" values for salt water in the formula $R=fAV^{1\cdot825}$, where

R=frictional resistance in lbs.;
V=speed in knots;
A=wetted surface in square feet.

Length in Feet:	50	75	100	200	300	400	500	700	900
f.	·0096	·00935	·0092	·00898	·0089	·00883	·00877	·00868	·0086

§ 10. **Extension to a Solid Surface.**—It is assumed that the values of "*f*" and "*n*" derived from the plank results are true for the curved and twisted surface of a ship. From a reference

to Chapter II. it may be seen that this cannot be strictly true. Along the greater part of the side of the ship there is a general excess of speed due to the stream line action, and a corresponding defect of speed at each end. It is very probable that the increased skin resistance due to this excess of speed more than counterbalances the reduction of resistance due to the lower speed at the end. In order, therefore, to obtain from plank results a correct estimate of the skin resistance of a ship, the calculation should be made using a speed slightly greater than that for which the estimate is required. Experiments with models at the slowest practicable speeds generally give a resistance exceeding that deduced from results for a plank of the same length and area by an amount varying from 5 to 20 per cent., as given in Table 3. Although some portion of this excess is due to wave-making—as even at these low speeds some wave-making exists—there is a strong probability that a large portion of it is due to the estimate of friction being below the truth for the reason already given. An analysis of the stream velocities for the forms given in § 5 shows that the *mean* velocity of the stream past the form exceeds the uniform velocity of the stream by the percentages given in Table 3. These apply to a very thin layer in contact with the form, and would be somewhat lower if taken over a thick layer, such as the frictional belt of any ship. The similarity of these figures and those given for the models gives good reason for increasing the velocity in making an estimate, the increase depending on the fulness of the form.

Rankine proposed to take account of this increase of velocity amidships by using in place of the true wetted surface an "augmented surface." This was intended to represent the *plane* area, which when moving at the same velocity would have the same frictional resistance as the ship—*i.e.*, it exceeded the actual wetted surface by an amount dependent upon the obliquity and change in velocity of the stream lines against the ship. Since the level lines and diagonals of the ship are not unlike trochoidal curves, Rankine chose such a curve to obtain the value of this augmented surface. In this case he found that approximately

$$\begin{pmatrix}\text{the augmented}\\ \text{surface}\end{pmatrix}=(I+4\sin^2\theta)\begin{pmatrix}\text{mean girth}\\ \text{of ship}\end{pmatrix}\begin{pmatrix}\text{length}\\ \text{of ship}\end{pmatrix}$$

where θ is the greatest obliquity of the trochoidal curve to the fore and aft line, or the mean obliquity of the level lines of a ship.

This augmented surface was proposed by Rankine for calculating the indicated horse-power. The formula, however, is not accurate for forms with much parallel body, and in any case the method does not take account of the variation in the proportion of wave to frictional resistance, which may be considerable. Mainly for these reasons it has never been largely used, but the formula serves to show that, so far as *frictional* resistance is concerned, increase in fulness carries with it a slight increase in mean rubbing velocity between the streams and the ship, which can be approximately allowed for on the frictional resistance by assuming either an increase in velocity or an increase in wetted surface in making the calculation from plank results.

TABLE 3.

Increase of Skin Friction due to Form.

Form.	A	B	C	D	G	Fine Battle-ship.	Merchant Vessel.		
							Fine.	Me-dium.	Full.
Prismatic coefficient	·50	·55	·65	·72	·76	·60	·66	·75	·82
Mean increase of V^2 of layer in contact with stream form	·128	·160	·185	·193	·225				
Mean excess of measured resistance over skin resistance calculated from W. Froude's results	—	—	—	—	—	·10	·10	·135	·21

As a check upon the figures given in the table experiments have been made with a given form in both air and water. The model tested in the water was 16 feet in length. The form had fine ends, and at low speeds wave-making was practically absent. At these speeds the resistance was found to be 10 per cent. higher than

that for a plank of equal length and area deduced from Froude's results. This result was obtained with paraffin, varnish, and red-lead surfaces. The model tested in air was 3 feet long, made symmetrical about the water plane, so that each half represented the under-water body of the ship. This compelled the streams at the load water plane to remain in that plane, wave-making being eliminated. But at low speeds in the water the stream lines are not appreciably affected by the wave-making, and the air model results should apply without correction to the water model. When the results were reduced to the equivalent speed and resistance for the 16-foot model in water the latter were found to be 1 per cent. lower in value than the resistance found by actual experiment with this model in water. This agreement between the two results, and the general likeness of the experimental and theoretical results given in the preceding table, show that the form affects the skin resistance, and that the true skin resistance is greater than that for a plank surface of equal length and area.

§ 11. Extension to Higher Speeds.—The results are extended to speeds above 8 knots by assuming that the "n" value does not alter at these higher speeds. In the Froude experiments this is practically the case for all speeds above 2½ knots. The only frictional experiments made at really high speeds are those of Dr. Stanton with water in a pipe ½ inch in diameter. The value of "n" in these experiments remained the same (1·775) throughout a velocity range of 2·6 to 100 feet per second. The similarity of flow in a small pipe and around a ship is very small, but as the resistance in both cases is of the same type this result at any rate supports the above assumption.

§ 12. Stanton's Experiments.—Osborne Reynolds found that water flowing along a pipe at low speeds moved in straight lines, but that at a certain critical speed this motion broke down and became sinuous or eddying.

If D is the diameter of the pipe and V the velocity of flow, eddying begins for all pipes at a definite value of $V \times D$. Further,

he found that for all similar pipes, if the speeds were such that $(V \times D)$ had the same value for each pipe, the frictional resistance per unit area of each pipe was in the ratio of the square of the speeds.

Dr. Stanton has amplified these results considerably, and has verified by experiment the formula * given by Lord Rayleigh for frictional resistance. This formula is as follows :—

Resistance per square foot

$$=R=\tfrac{1}{2}\times w\times V^2\times k,$$

where

k is a function of the variable $\left(w\times\dfrac{V\times l}{\mu}\right)$, l being a measure of length or dimension ;

μ the coefficient of viscosity of the fluid ;

w the density of the fluid.

This formula is mainly of use in obtaining resistances in air at low speeds from experiments with small models in water, or *vice versa*. Thus, if the value of $\dfrac{wVl}{\mu}$ is made the same in air as in water, then the value of $\left(\dfrac{R}{\frac{1}{2}wV^2}\right)$ is the same in each fluid. The value of $\dfrac{\mu}{w}$ for air is approximately thirteen times that of water. Hence, if it is required to obtain the resistance of an air balloon at speed V, a model of $\dfrac{1}{n}$th the size can be tested in water at speed $v=\left(\dfrac{V}{13}\right)n$, since at this speed $\dfrac{wVl}{\mu}$ is the same in air and water.

At speeds which bear the above ratio to each other, the resistance of the model in water R_W, and the resistance of the balloon in air R_A, are connected by the following formula :—

$$R_A=\frac{w_a}{w_w}\left(\frac{VL}{vl}\right)^2 R_W=\frac{R_W}{4{\cdot}7}.$$

where

w_a and w_w are the densities of air and water,

L and l are dimensions of balloon and model.

* Report of Advisory Committee for Aeronautics, Vol. I.

For high speeds in air the method is of no use as it requires too large models, and recourse must be had to Zahm's experiments on the friction of boards in air.

§ 13. Calculation of Immersed or Wetted Area.—A very fair estimate of the wetted area of a ship may be obtained by measuring the half-girths from keel to the load-line on the body plan, integrating by means of Simpson's rules, and adding 1 per cent. to allow for the curvature of the surface. A more accurate method is to increase the half-girths at each station to allow for

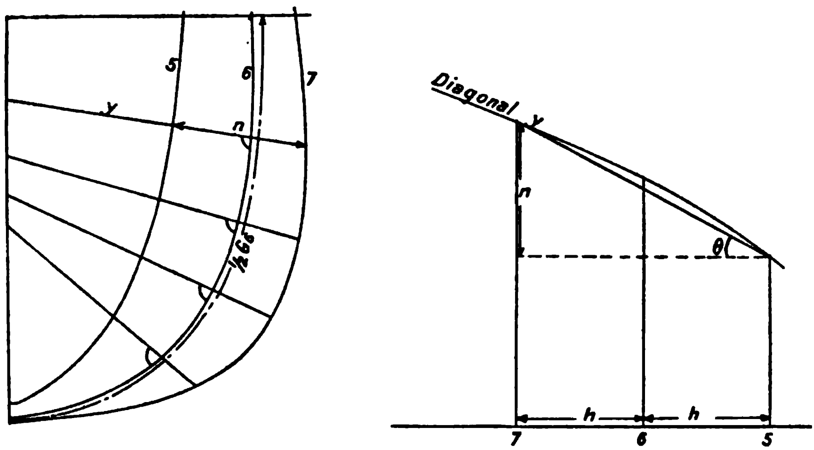


FIG. 6.

the mean inclination of the surface at that station and to integrate these modified half-girths as before, no allowance being made in this case for curvature of surface.

This modified girth is obtained as follows :—

In Fig. 6, which represents the body and half-breadth plan of a ship, the length of plating per foot run along any diagonal y between stations 5 and 7 is given by $\frac{1}{\cos\theta}$, where θ is the inclination of the diagonal line to the longitudinal axis of the ship. If G_6 is the wetted girth of station 6, and $(\cos\theta)_m$ the mean value of $\cos\theta$ for all the diagonal planes at this station, $G_6(\sec\theta)_m$ is the actual area of plating per foot run at station 6.

Cos θ is given by

$$\cos\theta=\sqrt{1+\tan^2\theta}.$$

and

$$\tan\theta=\frac{\text{area section 7}-\text{area section 5}}{2h\times G_6}.\ ^*$$

These areas of sections are known from the displacement calculation, or can be obtained by planimeter.

The work is done in tabular form as follows:—

Station Number.	Area of Section.	Difference of Areas.	$(\text{Tan }\theta)_m$.	$(\text{Sec }\theta)_m$.	Girth.	Modified Girth.	S.M.	Product for Wetted Area.
5	A_5	A_6-A_4	—	—	—	—	$\frac{1}{2}$	
6	A_6	A_7-A_5	$\frac{A_7-A_5}{2hG_6}$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	G_6	$G_6\sec\theta$	2	$2G_6\sec\theta$
7	A_7	A_8-A_6	—	—	—	—	$\frac{1}{2}$	—

Formula for Wetted Area.—Where time or circumstances do not permit of the wetted area being calculated in the manner described above, one of the following formula can be used.

Denny's Formula (Wetted Surface):

$$S=L(1\cdot7D+\beta B);$$

Froude's Formula (Wetted Surface):

$$S=(\Delta 35)^{\frac{2}{3}}\left(3\cdot4+\frac{L}{2(\Delta 35)^{\frac{1}{3}}}\right);$$

or

$$S=L(2\cdot0D+\beta B).$$

The former are applicable to fine vessels of ordinary form; the latter is more accurate for large block values or for extreme forms such as shallow draft vessels.

* Since $\tan\theta=\frac{n}{2h}$ and the mean value of $\tan\theta$ at section 6 is the mean value of all the normals between sections 5 and 7, *i.e.*,

$$\left(\frac{\text{difference of areas of these sections}}{\text{girth of section 6}}\right).$$

§ 14. Power absorbed by Skin Friction.—This can be estimated from the salt-water constants given in Table 2. Considering a ship 300 feet in length, the resistance R_f due to skin friction is

$$R_f = \cdot 0089 \times S \times V^{1 \cdot 825} \text{ lbs.}$$

and E.H.P. due to skin friction

$$= R_f \times V \times \frac{101\frac{1}{3}}{33000} = \cdot 0000273 S V^{2 \cdot 825},$$

S being calculated from either of the formula already given.

In terms of Froude's constants this formula becomes

$$\text{H.P.} = \cdot 0003 \, S \times \Delta^{\frac{2}{3}} \times V^{2 \cdot 825}.$$

The constant varies slightly with length owing to the variation of "f" in Table 2.

The above formula neglects the effect of form upon the result, and the E.H.P. should be increased by the percentage given in § 10 in order to arrive at an accurate value for the power actually used up in skin friction and form resistance.

TABLE 4.

Type of Ship.	Length in feet.	Block Co-efficient.	Speed in Knots.							
			10.	12.	14.	18.	20.	22.	30.	35.
Turbinia .	100	·50	61	—	—	—	43	—	53	54
Destroyer .	220	·50	83	74	—	—	46	—	43	44
Old-type battleship.	400	·65	75	—	—	55				
Cruiser :										
Small .	350	·51	87	—	—	—	—	48		
Moderate	450	·48	83	—	—	—	—	66		
Modern .	600	·48	83	—	88	—	65 at 25 knots			
Passenger steamer .	400	·57	86	—	—	—	55			
Cargo-passenger .	400	·71	73	72	54 at 15 knots					
Cargo boat .	400	·67	78	72	64					

The above figures are to be used with the formula given above, and do not include the effect of form upon skin resistance.

At very low speeds skin friction absorbs practically all the power required for towing the ship, but as wave-making grows it becomes of less importance. Table 4, p. 28, gives the percentage values of the whole tow-rope power which is due to friction for a number of vessels at different speeds. It will be seen that this percentage varies from about 85 at low speeds to 45 at high speeds, and the figures show the importance of a clean bottom which means a low frictional coefficient.

§ 15. Fouling.—Almost invariably ship trials are made with the bottom freshly painted with one form or another of anti-fouling composition. This is the condition which most nearly represents model experiments, as a varnish or red-lead surface gives practically the same result in the model as a paraffin surface. It may be generally assumed that so long as the paint is smooth, not lumpy, and above all not gritty, it will have about the same frictional value.

But all paints do not have the same anti-fouling properties, or rather they have not the same capacity for retaining a smooth surface. The priming and protective coats as well as the anti-fouling should dry and harden quickly, and the last coat should be free from grittiness or hard lumps. A paint which has good anti-fouling properties but peels off when exposed to air and water is not good. On the other hand, ever such a smooth paint which is deficient in anti-fouling properties is practically useless except as a possible protective to the hull.

The effect of fouling upon a ship's resistance can be very great. The reduction of speed caused by it on a fixed horse-power depends upon—

(1) The proportion of skin resistance to the total.

(2) The rate at which the total resistance is varying in terms of the speed; the higher this rate the less the reduction of speed.

(3) The normal slip of the screw. A decrease in speed of the ship means a higher slip ratio for the screw if it maintains the same revolutions, and in most cases this means loss of efficiency in the screw.

It follows that fouling is more important at low, *i.e.*, non-wave-making, speeds than at high speeds. It is difficult to define the effect of any degree of fouling, but many cases are on record of ships losing ½ to 2 knots simply because of the foulness of their bottoms. An examination of Mr. Froude's plank results shows that even the roughness of calico nearly doubles the resistance, and it would require but very little growth of weed or barnacle to give worse conditions than the calico. Experiments performed in the Spezia tank are stated to have shown that the resistance of a surface covered with incrustations is five times that of a freshly painted surface.

CHAPTER IV

EDDY-MAKING

§ 16.—It has already been stated that the frictional character of the water affects the resistance by the production of eddies. But eddy-making may be set up in other ways than by the friction of the surface of the ship and the water, and the main cause of such eddy-making is abrupt change of form, either as a too rapid change in shape or as a blunt end to any under-water feature such as shaft brackets, rudder, etc. The loss due to the change of form is very insidious. As the ship passes through the water, the pressure at every point in the water is continually changing. The greater the rate of change of pressure at any point the more sensitive becomes the stream line motion, and in a frictional fluid such as water the more liable is it to break into eddies at these parts.

At the after-body of the ship the streams near the form flow into a region of increasing pressure, causing a consequent drop in velocity of the particles. But if for any reason the particles are unable to give up sufficient energy of motion to balance the increase in pressure, they will break away from stream line motion and set up eddies or whirls. Even in a perfect fluid such eddies are possible, and in a viscous fluid such as water there is a much greater liability to it owing to the frictional action of the ship's surface. When such eddies are formed they are lost in the surrounding fluid, being left behind as the form advances, and there is a continual drain of energy necessary to create the turbulent stream formed at such parts. This eddy-making is more liable to occur wherever the curvature of the surface is changing quickly, and it is necessary to see that at such places as A in Fig. 5, where the U-shaped bow and stern sections are

merged into the squarer midship section, it is done gradually, and the bilge turn should be kept as easy as possible in the sections to avoid the rapid opening out of the stream lines indicated on this diagram, particularly in the after-body.

The energy lost in each whirling eddy thrown off will vary as the square of the whirling velocity, but the rate at which energy is lost will depend not only upon this, but also upon the size of the eddies and the rate at which they are shed. With regard to the latter we have no information. General consideration leads to the conclusion that the area of eddy-making will increase with velocity. The point of departure of the stream line from the form will probably be where the rate of pressure change is a more or less fixed amount, and will therefore tend to advance along the form in the direction of motion as the velocity increases. No law can, however, be obtained from such general reasoning, and to obtain one we must have recourse to experiment.

Experiments have been made in order to detect this eddy-making with two models A and B having 50 per cent. and 30 per cent. respectively of parallel body in their length, and a prismatic coefficient of run equal to ·638. There were approximately two and a half beams in the length of run of model A, which was shaped so that a stream line (which is in a diagonal plane, approximately), if it follows the form, must have a maximum angle of 22 degrees to the fore and aft-line. For the form B with longer run this angle was reduced to 16·5 degrees. The models were made so that the top-half resembled the bottom, and both halves were similar to the under-water portion of a ship up to its load-line. This confined the load-line stream to a plane, and when the model was deeply submerged wave-making was absent. But wave-making does not affect eddy-making to any great extent so far as we know, and it may be assumed that whatever is present under these conditions will be present when the form has a free surface. Both models were totally immersed in water contained in a long glass channel, with their keel lines parallel to the length of the channel. They were coated with condensed milk and the water allowed to flow past them at a

Horizontal View, Sternpost Vertical.

NOTE.—The L.W.L. is at the centre of the model as seen above.

View looking up from the Keel.

FIG. 7.—Eddy-making at Stern of Full Model (see § 16).

uniform velocity until the sides had been cleared by the water. Where the relative velocity was small, the milk remained behind, and the milkiness of the water at these parts enabled the extent of any eddies formed to be easily seen. Very emphatic eddy-making was found with form A, and Fig. 7 shows its extent. With form B there was practically no eddying except a slight amount near the water line plane. Resistance experiments with larger and exactly similar models showed that the resistance per ton was much greater for form A than for form B. But tests with other models showed that if the length of run was made greater than in B (by reduction of the parallel body) very little was gained by it. Further, an increase in the prismatic coefficient of the run from ·638 in model B to ·70, other things remaining the same as they were, caused a slight increase in resistance per ton at all speeds, and seemed to indicate that slight eddy-making was present with the fuller run. It seems probable, therefore, that in form B the lines are as angular as it is safe to make them if eddies are to be avoided.

A large number of similar experiments have been made at the National Physical Laboratory with balloon-shaped models having various endings. Eddy-making occurred at the rear of many of these, and the point at which these eddies commenced to form was found in almost every case to be where the tangent to the form was inclined at an angle of from 16 to 18 degrees to the axis of the form. This gives considerable support to what has been stated above relative to the formation of eddies behind ship-shaped forms, and leaves little doubt as to their cause.

This eddying at the stern of the ship has a threefold effect upon the general performance of the vessel.

(*a*) It adds very considerably to the tow rope resistance at all speeds.

(*b*) The majority of ships liable to this defect are slow single-screw ships, and if such eddies are formed the propeller must necessarily work in water which, over a portion of the disc area, is in violent turbulent motion. No propeller can work efficiently under these conditions, and there is a consequent loss of energy

due to the inefficient propeller action, which is cumulative to that under heading (*a*).

(*c*) The stability of the motion is affected if the eddying is on a large scale. As the eddies break away on one side or the other unbalanced forces are brought into play, and the helmsman finds it very difficult to keep the ship on her course.

§ 17.—The second possible cause of eddy-making is what is sometimes called "head resistance," due to such features as bilge keels, shaft and shaft brackets, rudders, etc. Most of these features, and particularly the bilge keel, are partially shrouded in the frictional belt of the ship, but those parts which project beyond this belt, such as shaft tube webs and brackets, should as far as possible lie in planes parallel to the stream line motion in their neighbourhood. If this is arranged the extra resistance due to them is merely frictional. But for some features, such as screw shafts in twin or multiple screw ships, this is impossible, and the added resistance due to the water sweeping diagonally across the shafts must be accepted.

There is not much difficulty in placing a bilge keel along the amidship portion of a ship so that it does not develop head resistance. At the ends where the form is changing more care is required. A study of the stream lines in Fig. 5 will give a good general idea of the direction the keel should take. These practical details will be considered later, only general considerations of the cause of head resistance being taken at present.

The head resistance due to a badly-placed keel plate may be judged from the following facts. The pressure on a plate inclined at an angle a (degrees) to its line of motion is given by—

$$P = kAaV_1^2,$$

and the power dissipated by

$$\frac{PV_1 \times 101\frac{1}{3}}{33000} = \cdot 00307kAaV_1^3$$

provided a is small, as it would be in the case of a bilge keel.

A is the area of one side of the two keels in square feet.

V_1 is the velocity of the water past them in knots. For

ordinary depths of keels this may be taken as ·5 V for long full ships, and ·6 V for short high-speed vessels, V being the ship's speed.

k is a constant which for a short deep plate* is ·2, and for a long shallow plate † varies from ·06 for angles up to 13 degrees to ·11 for angles from 20 to 37 degrees.

The effective horse-power lost through placing a pair of bilge keels of area 400 square feet at a mean angle of 2·5° from the best position becomes at 20 knots

$$\text{power} = \cdot 06 \times 400 \times 2\cdot 5(20)^3 \times \frac{101\frac{1}{3}}{33000} \times (\cdot 5)^3 = 183.$$

Allowing a propulsive coefficient of ·5 this means a wastage of 366 horse-power at the engine.

The head resistance on and the power consumed by the blunt ends of a shaft bracket arm or rudder can be calculated in the same way, using a "k" value of 3·2, and taking a as unity.

The extent and formation of eddies behind any such items depends upon the inclination of the plate to its line of motion. Ahlborn has shown by means of photographs that for angles below about 30 degrees (for a rectangular plate) there is only one main eddy system behind the plate, but for angles above 42 degrees there are two main whirls. The approximate formula given above is for small angles when only one main whirl exists, and holds for angles up to 13 degrees for a deep short plate and 37 degrees for a shallow long plate.

* Ratio of vertical axis to horizontal axis, 4 to 1.

† Ratio of vertical axis to horizontal axis, 1 to 3.

CHAPTER V

WAVES AND WAVE-MAKING

§ 18. Deep Water Waves.—In its entirety, this subject of wave genesis and propagation is a very complex one, demanding a mathematical knowledge of a very high order. The naval architect is, however, concerned mainly with the underlying principles, and these are comparatively simple. In giving them, the endeavour has been to make them intelligible rather than to give the complete treatment on which any conclusions or formulæ are based.

The most important wave is that propagated in deep and unlimited water, sometimes called an ocean wave. The commonly accepted theory of its propagation, after creation, is that known as the trochoidal theory. In this it is assumed :—

1. That the profile of an ocean wave is a trochoid, an assumption which is not far from the truth. A trochoid can be drawn in the following way :—A small hole is made at a radius r from the centre of a circular disc of radius R (Fig. 8). This disc is then laid on a flat sheet of paper with its circumference against a straight edge and rolled along it. As this is done a pencil point in the small hole will describe a wavy curve or trochoid on the paper. This curve will be more peaked in character the nearer the pencil hole is placed to the circumference of the rolling disc. When the disc has travelled a distance $2\pi R$ the curve will begin to repeat itself, and this distance is called the length of the wave or trochoid and $2 \times r$ is its height.

2. That each particle of water as the wave passes describes a circle in the vertical plane with uniform speed, each particle, no matter what its depth, making one revolution during the passage of a wave. The method of propagation of a wave can be seen from Fig. 8, in which the arrow heads indicate the direction

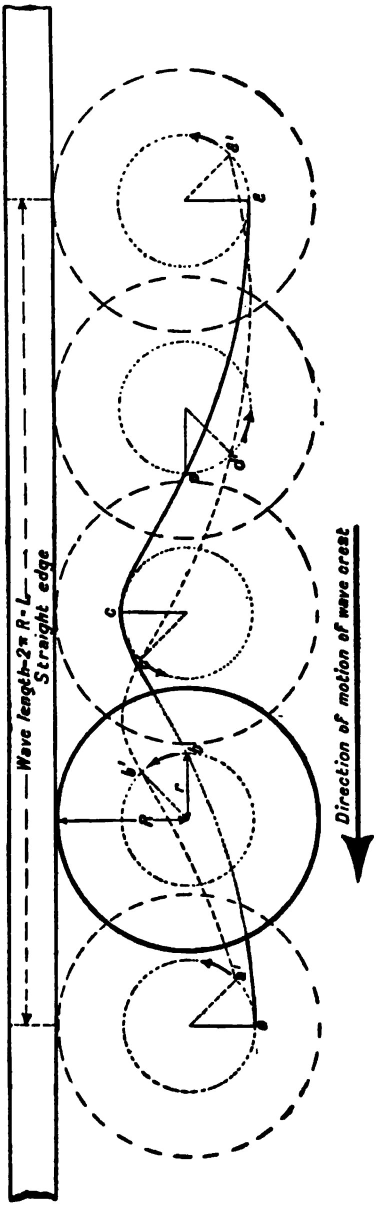


FIG. 8.—Trochoidal Wave.

of motion of the particles and the long arrow the direction of advance of the trochoidal wave. The particles $a, b, c, d, e,$ on the surface trochoid each turn about a centre fixed in space, and in unit time take up positions $a_1, b_1, c_1, d_1, e_1,$ and the dotted line through these points is the new position of the wave. It will be noticed that for such a wave the particles at the wave crest are moving in the direction of advance of the wave, and in the hollow they are moving in the opposite direction.

TABLE 5.

Wave Period (Seconds).	Length (Feet).	Speed of Advance.	
		Feet per Second.	Knots.
1·0	5·12	5·12	3·03
2·0	20·49	10·24	6·07
3·0	46·11	15·37	9·1
4·0	81·97	20·49	12·14
5·0	128·08	25·62	15·17
6·0	184·44	30·74	18·21
7·0	251·04	35·86	21·24
8·0	327·9	40·99	24·28
9·0	415·0	46·11	27·31
10·0	512·3	51·23	30·35
11·0	619·9	56·36	33·38
12·0	737·8	61·5	36·42
13·0	865·8	66·6	39·45
14·0	1,004·2	71·73	42·5

As the surface of the wave is exposed to the atmosphere it must be a surface of constant pressure, and for this to hold with the above assumptions, the resultant of the gravitational force and the centrifugal force acting on each surface particle must always be normal to the wave surface. This gives the following relation between the velocity of advance, the wave length and wave period :—

$$\left.\begin{aligned} V^2&=\frac{g}{2\pi}(L)=5{\cdot}123\times L \quad . \quad . \quad (1)\\ T^2&=\frac{2\pi}{g}\times(L)={\cdot}195\,L \quad . \quad . \quad (2)\end{aligned}\right\}\text{(see Table 5),}$$

where

V is the velocity in feet per second;

L is the wave length from crest to crest in feet;

T is the period in seconds, *i.e.* the time occupied by the wave in passing a fixed point.

The speed of a trochoidal wave is therefore dependent upon its length and is quite independent of its height, a most important fact in ship propulsion.

As by the initial assumptions the sub-surface particles move in precisely the same manner as the surface particles, it follows that lines of constant pressure, which before the passage of the wave were horizontal, as the wave passes will become trochoids having the same period and wave length as the surface particles, but different heights.

From the geometry of such a system of trochoidal lines can be obtained the following formula:—

$$h_1 = h\epsilon^{-\frac{2\pi d}{L}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$y = \frac{\pi}{4} \times \frac{h^2}{L} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

h is the wave height at the surface measured from crest to hollow;

h_1 the height of any sub-surface trochoid;

d the depth of the sub-surface below the mid-height of the surface trochoid;

y the distance between the mid-height of the surface trochoid and the surface of the water if the wave subsided.

The movement therefore decreases in geometrical progression at points whose depth below the surface varies in arithmetical progression, and at a depth equal to $\frac{1}{2}L$ the movement is only 4 per cent. of that at the surface.

From these equations can be found the potential energy (E_P) of the whole wave, *i.e.* the energy due to the water being slightly elevated above its still level.

$$E_P=\frac{w}{16}Lh^2-\frac{\pi^2}{32}w\frac{h^2}{L} \quad . \quad . \quad . \quad . \quad (5)$$

all dimensions being in feet and w being the weight of water per cubic foot.

The latter term can be neglected for all ordinary waves, and the equation for salt water becomes

$$E_P=4Lh^2 \text{ foot-pounds} \quad . \quad . \quad . \quad (6)$$

The energy represented by the circular motion of the particles throughout the fluid is equal to the energy of position, and the total energy in the wave (E) is given by

$$E=8Lh^2 \text{ foot-pounds} \quad . \quad . \quad . \quad (7)$$

per foot breadth of the wave measured along the crest line.

§ 19. Group Velocity.—Considering the conditions under which a group of such trochoidal waves can be propagated along the water surface, it is evident that as any wave travels along it must take with it that amount of energy which is necessary for its formation. But any particle of water in the wave system although moving in a circle does so with uniform velocity, so that its kinetic energy is the same at every instant during the passage of the wave. As the pressure also remains constant the only means by which such energy can be transmitted is by the potential energy of the particle as it rises and falls in its circular orbit. This energy is not stored up in the particle, but as every wave passes must be freshly acquired. As a single wave travels from end to end of the group, it finds water having the necessary circular motion, but if it is to travel without deformation the energy to raise the water particles above their still water positions must be transmitted with the wave, and this energy represents one half that of a wave. In other words, when each wave has moved forward one wave length, the energy of the group has moved forward half a wave length.

The front of a procession of waves entering undisturbed water, since it is only supplied from the rear with sufficient energy for its potential requirements, will rapidly decay unless an amount of energy equal to the kinetic energy of the wave (*i.e.*, the energy

due to the circular motion) is supplied from some foreign source. A ship which is making waves must supply this energy to the waves, and this dispersal of energy is one of the important items in a ship's total resistance.

Combination of Wave Systems.—If two systems of waves having the same velocity and length, and of amplitudes h_1 and h_2, travel in the same direction, with their crests distant from each other an amount d, the two will combine to form one uniform series of the same velocity and length, but an amplitude h_3, given by

$$h_3^2 = h_1^2 + h_2^2 + 2h_1h_2 \cos.\frac{2\pi d}{L} \quad . \quad . \quad . \quad (8)$$

Since the energy of the resultant system varies with

$$L \times h_3^2,$$

it can be seen this will vary with change in "d" value about a mean value depending on—

$$L(h_1^2 + h_2^2),$$

the amplitude of the oscillation being

$$2Lh_1h_2$$

This first term is merely the sum of the energies of the two systems taken separately, and by combining the systems with varying phase value d the energy of the group formed can be made greater or smaller than the total energy of the two taken separately—a point of great importance.

§ 20. Wave Generation.—The preceding theory assumes the existence of the waves and ignores the mode of generation, which, however, plays an important part in the question, when we come to deal with waves thrown off by a ship. It has already been shown that to create a procession of waves energy must be given to the front of the procession, and in the case of a ship this energy is obtained from the pressure changes produced in the water by the passage of the ship. Broadly speaking, these pressure changes are similar to those obtained by the consideration of a submerged form, and it may be assumed that so far as wave-making is con-

cerned a ship is similar to a travelling pressure disturbance of the character shown by Fig. 3, except in so far as this may be altered by the wave motion itself.

By supposing that the pressure areas P of Fig. 3 are concentrated at points at each end, Lord Kelvin in a deeply mathematical paper * has shown that each travelling point of pressure will generate a system of waves as shown in Fig. 9, *i.e.*, a series of transverse and slightly curved waves and the familiar oblique

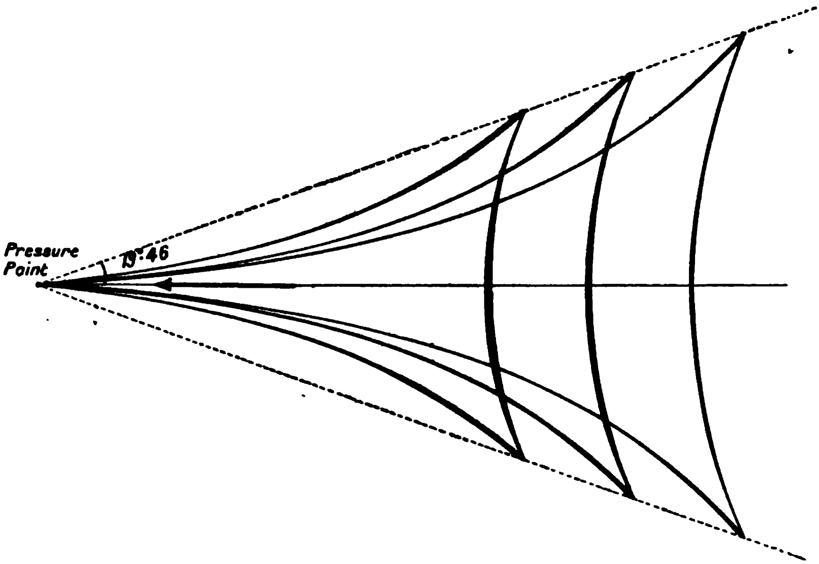


FIG. 9.—Wave Pattern produced by Travelling Point of Pressure.

waves with their outer ends terminating along a line inclined at 19°·46 to the path of the travelling point. When at some distance to the rear, the height of the transverse waves decreases inversely as the square root of the distance from the point. Although in a ship there is no such thing as a point of concentrated pressure, it is reasonable to suppose that if the pressure is dispersed over a small area the effect will be practically the same, and the solution then bears a closer resemblance to the ship.

One other interesting form of pressure disturbance has been

* "Mathematical and Physical Papers," Vol. IV.

treated mathematically, viz., the case of a disturbance of unlimited extent transversely and of the character of the curve for form C in Fig. 3. It has been shown that each crest of pressure will create a series of transverse waves, and that the resultant system will depend upon the distance apart of the pressure humps. It is also worthy of notice that in such a case, the wave height at high speeds decreases with increase in speed, unless the pressure disturbance increases in value.

These two theoretical cases show how the main systems of waves may be created, and although a ship is not the same as either, it has features common to both. Thus the wave pattern for any ship at all speeds consists of a train of transverse waves and two sets of diverging waves, one emanating from a little aft of the stem and the other from a little before the stern post. At low speeds for the ship these diverging waves, particularly the bow set, are far more noticeable than the transverse; but as speed is increased the latter become more prominent and appear at the side and rear of the ship as a series of waves of diminishing height.

§ 21. Diverging Waves.—These can be seen better by studying an actual ship than by the use of illustrations, and the reader should form the habit of observing the wave patterns of any of the ferry or other steamers around the coast and should carefully watch the change in wave pattern as the speed increases.

These diverging waves trail away from the bow at all speeds. They are also formed at the stern; but this system is not nearly so important as the bow system. When clear of the ship the highest points of the crests of such waves lie on a fairly straight line inclined to the ship's middle line at an angle a in Fig. 10. The crest lines themselves are inclined at approximately double this angle.

If V is the speed of the ship in feet per second the speed of the diverging waves normal to their crest lines is :—

$$V \times \sin\beta.$$

The distance d between consecutive crest lines when clear

of the ship and parallel to each other, measured normal to the crest line, is given by—

$$d=\frac{2\pi}{g}V^2\sin^2\beta \qquad . \quad . \quad . \quad . \quad (9)$$

The length of the diverging waves measured *along* their crest lines, created in unit time on each side of the vessel, is proportional to

$$V\times\cos\beta,$$

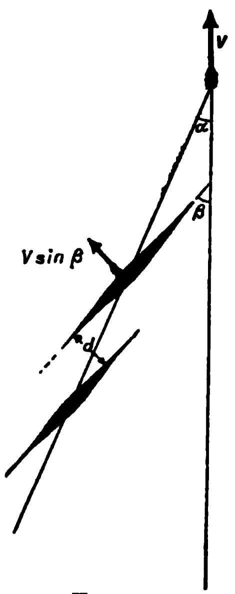


FIG. 10.

and if h is the height of the waves formed, the energy required for their formation is proportional to

$$(d\times h^2)(V\times\cos\beta),$$

which is equal to

$$kh^2\times V^3\times\sin^2\beta\cos\beta,$$

k being a constant.

Lord Kelvin's pressure point theory confines all the diverging waves at all speeds within two lines given by $a=19^\circ{\cdot}46$. The size and fulness and speed of the ship, however, affect these angles to some extent, and in Table 6 approximate values of β are given for various forms.

It will be seen that generally speaking the waves are more oblique the finer the angle of entrance and the lower the prismatic coefficient, but the form of the hull generally has some effect. This is particularly so as regards the first one or two waves near the bow. A well-rounded stem will create a fan-shaped bow breaker even though the angle of entrance is low. Or, again, a little hollow in the ends of the levels near the water line will give a much better and clearer system of bow waves than would be obtained if the levels were straightened out.* This improvement

* See, however, § 42 on Full-bowed Ships, and § 54 on Straight v. Hollow Lines.

in the diverging waves is accompanied by an appreciable reduction in resistance.

TABLE 6.

Angle of Obliquity of Diverging Waves.

No.	Type of Ship.	Half Angle of Entrance in Degrees.	Prismatic Coefficient.	$\frac{V}{\sqrt{L}}$.	Angle of Obliquity β in Degrees.
1.	Old cruiser .	23	—	·66	36
2.	Fine cruiser .	9·2	·41	—	27
3.	Cargo passenger steamer.	16·5	·672 (entrance only)	—	24
4.	Old-type battleship . .	20	·65	·85	30·2
5.	Battleship .	15·5	·65	·71 1·10	34 25
6.	Liner . .	8·5	·60	·69 ·89	30 25·5
7.	Torpedo-boat .	12·5	·66	1·27	19·5
8.	Destroyer .	9·0	·63	·67 2·1	21 11
9.	Motor launch .	5·5	—	2·59 3·14	5·75 10·5

	Inclination of Surface to Horizontal.	Immersion.	Angle of. Obliquity
Flat plate 1 foot wide, skimming on water at high velocities.	3°—6°	8 inches or less.	7°
	13°	15 inches	13°

The effect of speed upon obliquity can be seen from numbers 5, 6, and 8 above, but in the latter, part of the reduction is no doubt due to the bow rising out of the water at the higher speed, and only the lower and finer level lines are operating on the water.

The flat plate results have been given, as such a plate is working under much the same conditions as a skimmer, and shows the effect of too great an angle of bottom and too great immersion. Other results not given in the table show that immersion, although

having a moderate effect upon obliquity, is not so important as the angle of the bottom to the horizontal.

§ 22. Formulæ for Wave Resistance.—It has been stated in § 18 that the total energy of a wave is proportional to the product

$$\text{length (height)}^2.$$

A disturbance which is producing a group of such waves, in travelling two wave lengths, must supply the energy of one wave. The time occupied in travelling this distance is equal to

$$\frac{2L}{V},$$

L being in feet, V in feet per second. Hence the *rate* of dissipation of energy is proportional to

$$\frac{Lh^2}{\frac{2L}{V}},$$

or to

$$V \times h^2.$$

Professor Havelock has shown that a very wide pressure disturbance somewhat after the character of the curve C of Fig. 3 will produce waves whose amplitudes are given by :—

$$h = \frac{P}{V^2}\epsilon^{-4\left(\frac{V_1}{V}\right)^2} \quad . \quad . \quad . \quad . \quad (10)$$

where

P is the total pressure causing the disturbance,
h is the height of the waves,
V is the velocity in feet per second as above,
V_1 is a constant depending on the form.

These waves are two-dimensional, *i.e.* the motion and the variation of pressure are the same in any plane at right angles to the wave crests. This is not the case in a ship, as in the latter there is a comparatively rapid decrease of pressure transversely. But the formula serves to give a general indication of the effect of a travelling area of pressure.

For a fully submerged body $\frac{P}{V^2}$ would be fairly constant, and,

if it is assumed so for a ship on the water surface, creating waves whose amplitudes are given by equation 10, the rate of dissipation of energy becomes proportional to

$$\left\{N+M\cos\frac{2\pi d}{L}\right\}\epsilon^{-\frac{3}{2}\left(\frac{V_1}{V}\right)^2}\times V\times b.$$

N and M being constants depending on h_1 and h_2 of equation 8;

b is the breadth of wave created.

If we assume that the transverse waves are confined within the envelope of the diverging waves (this being true in the case considered by Lord Kelvin), the breadth b will vary with the angle a and will decrease as velocity increases. The bracketed term oscillates about a mean value, and the exponential term increases slowly at low speeds and then more rapidly as the speed is increased, the power curve having a point of inflection in it at the speed V_1.

This treatment can be extended to diverging waves, and, according to the assumptions made, interesting results may be obtained.

A simpler formula can be obtained by considering the heights of the transverse and diverging waves as varying with the water pressure, which varies with V^2. The same difficulty as before is met with, however, and some assumption must be made as to the relation of divergent and transverse waves. So far as the transverse waves are concerned the rate of dissipation of energy is proportional to

$$\left\{N+M\cos\frac{2\pi d}{L}\right\}bV^5.$$

From the expression in § 21 the power required for the diverging waves is proportional to

$$K\times h^2\times V^3\sin^2\beta\cos\beta.$$

K, M, and N are constants, as before, and h is the height of the diverging waves. By assuming h to vary as V^2 these two power terms can be gathered together in a comparatively simple formula.

It must be remembered in working with it that β is not independent of the velocity.

These formulæ, however, although giving a good idea of the growth of resistance due to the waves, are of no use in estimating work. The constants K, M, and N depend upon the form and are bound to vary with any change either in shape or dimensions. If by trial and error such formulæ are found which give a curve which approximates to a ship's trial results, it can only be used in other cases with the greatest caution, and only for ships of the same general type.

§ 23. Shallow Water Waves.—In the case of shallow water waves the particles move in ellipses instead of in circles as with the deep water waves, and their kinetic energy changes during the passage of the wave form. The velocity and wave length are connected by the following formula :—

$$V^2=\frac{g}{2\pi}\frac{b}{a}L \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where b is the vertical and a the horizontal axis of the surface ellipse. The group velocity is in this case somewhat greater than one-half the velocity of the individual waves.

The relative value of b and a depends upon the depth of water (h), and is given by

$$\frac{b}{a}=\frac{\epsilon^{\frac{4\pi h}{L}}-1}{\epsilon^{\frac{4\pi h}{L}}+1} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

When the depth h is small compared with the wave length L the above reduces to

$$\frac{b}{a}=\frac{2\pi h}{L} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the velocity of such a wave in feet per second is given by

$$V^2=g\times h \quad . \quad . \quad . \quad . \quad . \quad (14)$$

With V in knots, this becomes

$$V^2=11{\cdot}28h.$$

It must be carefully borne in mind that this only holds provided that h is small compared with L, a condition which is seldom reached so far as the naval architect is concerned.

It can be shown that the diverging waves set up by a pressure disturbance of small dimensions travelling into shallow water increase in obliquity as the water gets shallower. This increase in obliquity begins when the velocity becomes ·7 times that given by formula 14, and continues until this critical speed is reached. There is also a growth in the height of the transverse waves, which becomes most marked as the critical velocity is reached, when the wave travels at the same speed as the disturbance causing it, and all the wave-making is therefore concentrated in one large wave.

Beyond this critical speed only diverging waves can be created, and the obliquity of these decreases continuously with increase of speed until the deep water value (19°·46) is reached, when the speed is approximately equal to

$$3 \times \sqrt{gh},$$

or three times the critical velocity given by formula 14.

This growth of the transverse waves up to a certain speed and the absence of them above that speed has been noticed in many cases when a ship has steamed into shallow water, but the effect of the shallow water upon the divergent waves has never been noted.

CHAPTER VI

SHIP MODEL EXPERIMENTS

§ 24.—The tow-rope resistance and effective horse-power of a ship can be determined from the results of resistance experiments with a model of exactly similar under-water shape. The late William Froude, to whom the science of naval architecture owes so much, was the first to demonstrate the feasibility of this, and to him is due the law of comparison connecting the speeds and resistances of ship and model when both are creating similar wave and stream disturbances which differ from one another only in scale. Every nation of any maritime importance possesses an experiment tank for the determination of ship resistance from that of models, and, although differing in detail, the mode of experiment and method of deducing the power for the ship is in almost every case the same as that adopted by Mr. Froude in his tank which was erected at Torquay in 1870. A list of these tanks with their principal dimensions is given in Table 7.

The models used are really hollow shells, occasionally made in wood, but usually in paraffin wax, as the latter is less costly and more easily varied in shape, and a model in wax can be made more quickly than one in wood. They vary in length from 10 to 20 feet, and in thickness from 2 inches in a full form to $\frac{5}{8}$ inch in a light, fine form, such as a destroyer. The reader is referred to a paper read before the Institution of Naval Architects in 1911 for a full description of the apparatus for making these models. Briefly, the models, which are cast slightly larger than required, have a series of grooves cut in them corresponding to the ordinary level lines of the ship. Both sides of the model are cut at the same time so that it shall be perfectly symmetrical. The cutting machine consists of two tables, one to carry the model and the

other to carry a plan of the level lines. A tracer working on the latter table is connected to the frame carrying one of the cutters by a pantograph lever, so that the position of the cutters relative to the model is exactly the same as that of the tracer relative to the level line being cut. The wax between the grooves is removed by hand afterwards.

TABLE 7.

General Dimensions of Experiment Tanks.

Name.	Length having full Depth of Water in feet.	Breadth on Water Surface in feet.	Depth of Water in feet.	Area of Cross-section in square feet.	Maximum Velocity in feet per second.
Berlin	479	34	11·5	265	23
Clydebank	400	20	9·5	180	16
Denny's	275	25	10		
Hamburg	1,088	26·2	21·0	—	36 (est.)
Haslar	400	20	9	170	16
Japan	450	20	12	235	20
Michigan	300	22	10	200	13
Paris	528	32·8	14·0	326	15
Uebigau	289	21·4	11·3	200	16
Vicker's	420	20·0	—	—	—
Vienna	550	32·8	16·4	—	24
Washington	384	44·6	14·75	418	30
William Froude	494	30	12·5	360	25

As a rule the models are tested without rudder, shafting or bilge keels. This enables the relative merits of different forms to be compared, and eliminates any error due to the doubtful application of the law of comparison to the resistance of appendages. The models are towed along a waterway or canal, and by means of an extremely sensitive dynamometer the resistance is accurately measured for a large range of speeds. The speed of each experiment is obtained in much the same way as that of a ship when on a measured mile. Along the waterway there are "posts" every 20 feet. A continuous record of time is taken on the diagram by means of a pen giving a mark every half-second, and

a similar mark is obtained by another pen when the model passes each of these "posts." Thus time and distance are recorded, and so the speed may be obtained and its constancy can be checked from moment to moment. A skeleton diagram of the resistance dynamometer is shown in Fig. 11, which largely

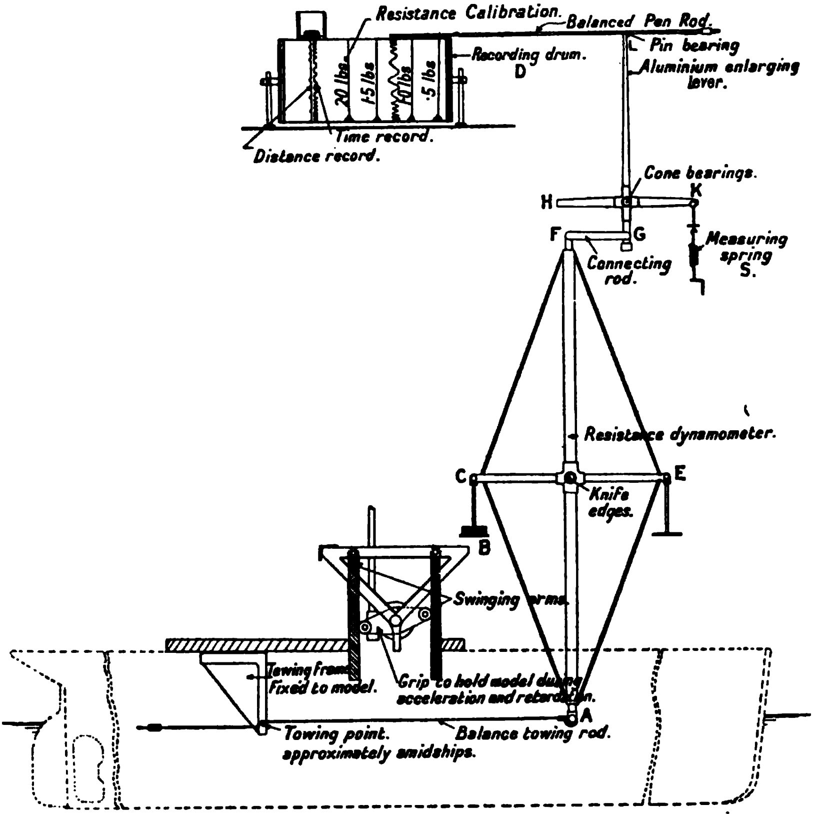


FIG. 11.—Ship Model Towing Apparatus.

explains itself. The resistance or pull at A is taken partly by weights on the pan B, and partly by the elongation of the spring S, this elongation being recorded on the revolving drum D. The dynamometer lever $ACEF$ and the recording lever $GHKL$ are both exactly balanced, so that their centres of gravity are on their lines of suspension, and when they take up an inclined position

no error is introduced by any movement of their centres of gravity.

§ 25.—As the skin friction resistance depends in part upon the length of model, a correction has to be made in the resistances in applying them to the ship. This is done by the use of the results of either Froude's or Tideman's frictional experiments. This correction can be made in one of two ways. Both require a knowledge of the wetted surface of the form. In the first method a separate calculation is made for the frictional resistance of the model at various speeds, and these resistances are deducted from the measured resistances of the model. The residual resistance found in this way varies according to Froude's law. Thus, if at speed v in feet per minute the residual resistance of model is found to be r pounds, then for a ship l times the linear size of model, the residual resistance at speed

$$V = v\sqrt{l} \text{ feet per minute}$$

will be $r \times l^3$ lbs. in fresh water, since the experiments are made in fresh water. Or in the usual ship units, at speed

$$V = \frac{v}{101 \cdot 3}\sqrt{l}$$

in knots the residual resistance is

$$= r \times l^3 \times \left(\frac{64}{62 \cdot 5 \times 2240}\right) \text{ tons.}$$

In order to get the total resistance for the ship, its frictional resistance must be calculated and added to the above. For this the wetted surface is known, being l^2 times that of the model, and the resistance can be calculated using the coefficient for the length of ship as given in Table 2.

The second method, which is more commonly adopted in experimental practice, is to calculate the difference in the value of $\frac{r_f}{v^2\delta^{\frac{2}{3}}}$ and $\frac{R_f}{V^2\Delta^{\frac{2}{3}}}$ for model and ship respectively, at corresponding speeds v and V, where r_f and R_f are the frictional resistances at these speeds. This amounts to the same thing as the above, but has

the advantage of giving the correction for ship in a "constant" form. The above difference can be worked out for ships of several lengths, and in this way correction curves can be drawn on the plotting of the model C values, and the difference in ordinate value between these correction curves and the model C curves can then be treated by the law of comparison in the usual way.

§ 26.—In order that the results so obtained shall be free from any doubt it is essential

(1) that similar models made from the same set of lines shall always give the same result within the usual limits of error;

(2) that tests made on a full-sized ship shall agree with the results of the model experiments.

With regard to (1), tests made with three different models at the National Physical Laboratory, all to the same set of lines but tested at different times, have differed by not more than 1½ per cent. A more searching test is the comparison of horse-power curves for the same ship obtained from model experiments in different tanks. This has been done with several models. The results obtained at Haslar and at the National Physical Laboratory have agreed within 1½ per cent.; results obtained at Clydebank, Dumbarton, Washington, and the National Physical Laboratory have all agreed within 3 per cent. throughout the practical range of speed.

With regard to (2), several tests have been made at different times with favourable results. The chief of these is the classical research * of the late W. Froude on the *Greyhound* in 1873. This vessel, of dimensions—length, 172·5 feet; beam, 33·1 feet; displacement, 1,050 tons, was towed from the end of a 45-foot spar projecting over the side of the *Active*, so that the bow of the *Greyhound* was 50 feet beyond the stern of the *Active*. This device was adopted in order to avoid the wake and wave effect of the towing ship. The speed through the water and the tow-

* "On Experiments with H.M.S. *Greyhound*," Trans. I. N. A., 1874.

rope pull were both recorded. The air velocity was also measured in each case in order that a correction for the air resistance could be obtained, additional experiments being made for this purpose. The curve of towing force obtained in this way was then compared with that estimated from tank experiments with a model one-sixteenth the size of the ship.

This estimate for the ship was made in the manner already described, using for both model and ship, frictional coefficients which were correct for a smooth varnished surface. The measured pulls on the ship were slightly in excess of those deduced from the model. The ship's bottom was coated with copper, which was not new and was probably deteriorated by age, and its frictional value would therefore be somewhat greater than that of a smooth varnished surface. In order to bring the estimated and measured resistances into agreement it was found necessary to make one of two assumptions, viz :—

(*a*) either the skin friction of the ship was 30 per cent. greater than that of the model; or

(*b*) one-third of the ship's surface had a resistance equal to that of calico and the remainder that of a varnished surface.

Either of these assumptions will bring the estimated and measured curves into exact agreement at all speeds, and for the reasons given such an increase appears to be reasonable.

In 1883 Mr. Yarrow published* the results of similar trials made with a 100-foot torpedo-boat at a displacement of 40 tons, and with a model of the same in the Haslar experiment tank. The effective horse-power estimated from the model was found to be 3 per cent. less than the tow-rope horse-power measured on the vessel, which, however, carried slight bossings, shafts, and A-frames, not fitted to the model. This comparison was obtained over a speed range of 11·7 to 15·0 knots, the vessel being designed for 22·5 knots.

It is worthy of note here that carefully analysed steam trial results show the same characteristics in the curve of resistance for

* Trans. I. N. A., 1883.

the ship as are obtained by the model experiments. We have every reason, therefore, to be confident that model experiments are a sure guide to both the power required to drive a ship at a given maximum speed and the relative resistance value of different forms.

Although experiment tanks exist mainly for such work as that just described, this is by no means their only use. The position of the wave surface on a ship's side is very important in the case of all paddle steamers, and can be accurately and quickly obtained by observations with a model over the desired range of speed. The general effect of depth of water, canal banks, and many other such problems may be readily investigated in a tank.

CHAPTER VII

DIMENSIONS AND FORM

§ 27.—It is proposed in this and the succeeding chapters to consider only the tow-rope, or, as it is more properly called, the effective horse-power of the ship. The relation of this to the indicated or shaft horse-power will be dealt with later on. This separation of the question of the means of producing the necessary thrust from the problem of how to reduce that thrust to a minimum is legitimate and proper, and renders the designer's work somewhat less intricate.

Speaking broadly, a design is usually required to attain a certain speed with a specified load. In many cases there are restrictions as to draught of water, and the possible beam or length may occasionally be dependent on docking or canal conditions. These considerations, although important, do not come within the scope of this work, and the designer must himself weigh the advantages in propulsion which he may gain by certain changes, against the possible disadvantages in other directions.

There are so many variables involved in the form of a ship that to lay down a general law of resistance due to form which shall be applicable to all cases is impossible. The best that can be done is to consider each of these variables separately, or, where necessary, in conjunction with others, and to define the effect of variations in them upon the ship's resistance. The results are probably more useful in this form than in any other, as the problem before the designer is generally that of settling which of several variations of a parent form he shall adopt. If, as is often the case, he has a fair knowledge of the power required in the parent ship form, he can then make a good estimate of the effect of any change introduced.

It may be said that generally for given displacement and length, provided that the form is a fair one and no serious eddy-making takes place, the resistance of a ship at any speed is to a very great extent determined by

(*a*) *The shape of the curve of cross-section area, including the longitudinal or prismatic coefficient;*

(*b*) *The extreme beam;*

(*c*) *The normal water line, particularly that of the fore body.*

If the length may be considered as a variable, this is largely dependent upon the service speed of the ship. Unfortunately propulsion is not always the first consideration in the mercantile marine, but nevertheless the question of having sufficient length in both entrance and run for economical running is of great importance. This is particularly the case for cargo-passenger steamers where moderate speeds have to be obtained with high block coefficients.

§ 28.—For all moderate and low-speed vessels there is for any given maximum speed *a certain length of entrance and of run which is necessary to avoid abnormal wave-making and eddy-making resistance.* Wave-making resistance exists at all speeds, and in part cannot be avoided except possibly in submarines. At low speeds it is almost entirely due to the creation of diverging waves. These are of short length, and with the exception of the bow breaker never of any great height, and are largely dependent upon the form of the ship at each end near the water line. They do not depend to any measurable extent upon the length of the ship, whether this exists in the form of parallel body, entrance, or run. At higher speeds, as transverse waves are formed, length becomes of greater and greater importance.

This can be seen by reference to Fig. 12, which is a typical curve of resistance plotted in the form of C to a base of K (see § 2). This curve is generally flat over a considerable portion of its length, indicating that the resistance is varying

with the square of the velocity, but it has humps more or less marked in character, and ends at the high velocities with a quick bend upwards. As the frictional resistance increases at a lower rate than V^2, this abrupt increase must be due to wave-making. This receives the strongest confirmation in the rapid growth in the size of the waves formed as these speeds are reached.

Above all things the designer has to line out the form so that it shall escape this abnormal rise in resistance. The speed at which this rise begins varies with different ships. It is practically independent of beam, draft, and midship section shape. If the length is increased, keeping the form otherwise the same, this critical speed will increase with $(\text{length})^{\frac{1}{2}}$. For ships without parallel body this maximum speed for economy is given approximately by the formula :—

$$V = 1{\cdot}05\sqrt{L}$$

but it naturally depends somewhat upon the shape of the ends of the vessel. This formula gives lengths slightly less than those laid down

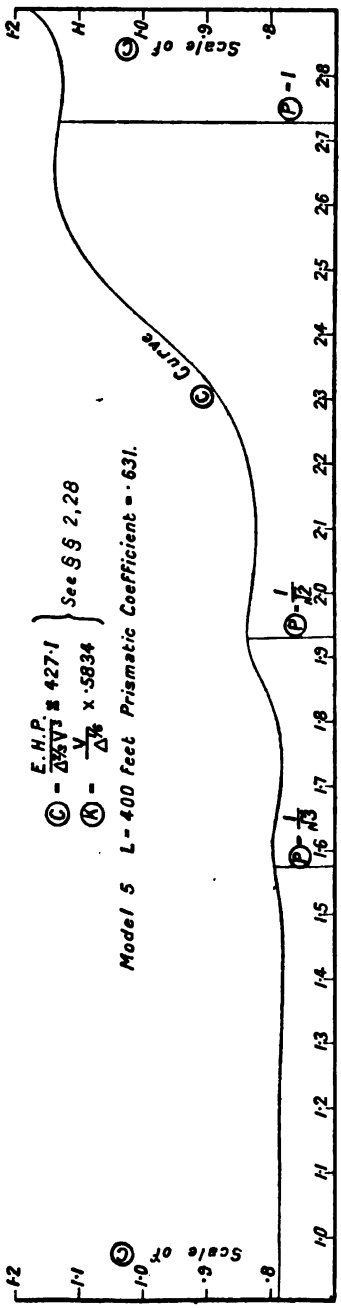


FIG. 12.—Typical (C) Curve.

by Scott Russell, but these are known to be too long particularly in the entrance. With ships whose prismatic coefficients exceed about ·55 the above formula is not very accurate. The wave systems created by both the bow and the stern of any ship depend upon the pressure disturbances produced in the water, and the resultant system formed by the superposition of the two systems will vary according to the relative positions of the humps and hollows in the pressure curve (see § 19). It has been shown in the section on stream lines that the magnitude and relative positions of these humps and hollows in the pressure disturbance around the ship depend upon two things, viz. :—

(*a*) the length of the ship, and
(*b*) its prismatic coefficient.

It follows therefore that the speed at which wave-making becomes serious will also depend upon these two factors, and not upon one only.

The analysis of many ship trials and model experiments has shown that the speed at which good or bad interference of bow and stern systems takes place depends very largely upon the simple product :—

$$\text{length} \times \text{prismatic coefficient or } (P \times L)$$

and that the critical speed of any ship is given by :—

$$V = 1{\cdot}34\sqrt{\frac{P \times L}{n}}$$

The integer n takes account of the number of wave *crests* between the bow and the stern systems. When n is one, there is one wave crest amidships between the bow and stern systems. When n is two, there are two wave crests between the first crest of the bow wave system and the first wave crest of the stern wave system, and so on.

At low speeds, when n is 4 or 3, or in the case of fine ships, 2, the humps in the resistance curve are generally too small to be worthy of notice, but with increase of speed it will be found that the value

of $\frac{\text{resistance}}{(\text{velocity})^2}$ will ultimately increase rapidly and this hump will be followed by a shorter or longer range of speed over which $\frac{R}{V^2}$ remains fairly constant or may even drop. The preceding formula gives the speeds for the *flat* at the top of the hump.

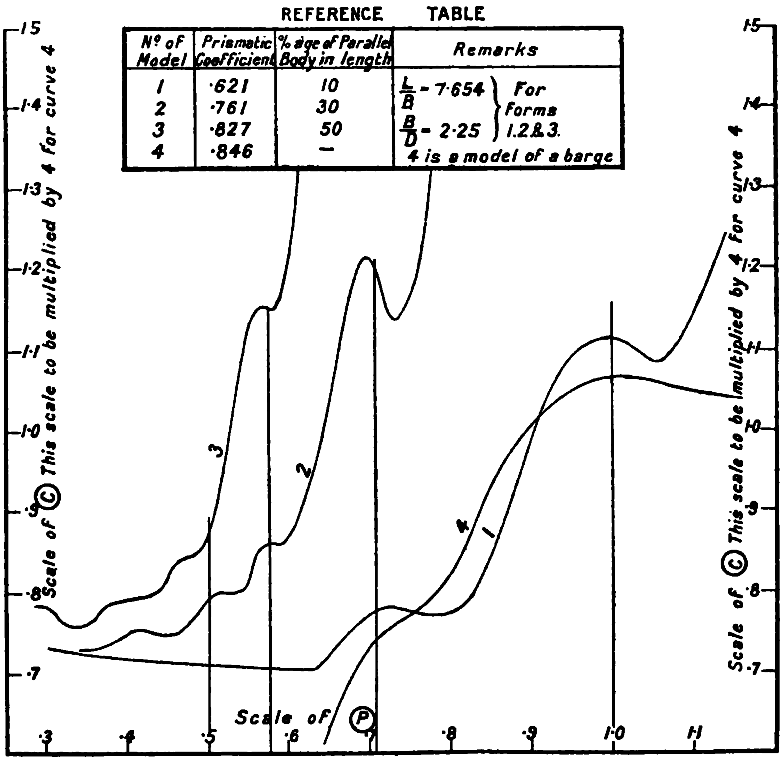


FIG. 13.—"Constant" Curves for Typical Ships.

The value of n to be taken depends upon the dimensions and speed. This can be seen from Fig. 13, which shows the "constant" curves for four typical models, all of which curves have large humps in them. The long, full form has a marked hump and flat at $(P) = \frac{1}{\sqrt{3}}$, and it could not be worked efficiently above

$(P)=\cdot 5$. The fine form number one has humps followed by flats at $(P)=\frac{1}{\sqrt{2}}$ and 1, and could be worked very efficiently to $(P)=\cdot 63$.

The designer's business is to keep clear of these humps, or, if this is not possible, to suppress them. The latter can be done in either of two ways. For vessels having little or no parallel body, and speeds given by $(P)=\frac{1}{\sqrt{2}}$ or 1, a fine bow prismatic with the entrance water line as long as possible is good. But for low-speed vessels this involves the sacrifice of too much displacement, and in these the humps can be largely eliminated by the adoption of straight lines at the ends. This causes the value of $\frac{R}{V^2}$ to increase more steadily with speed, and partially eliminates both hollows and humps. The improvement at one speed, therefore, is obtained by the sacrifice of a good result at another, and a designer must know the type of ship with which he is dealing in order to avoid a bad result at the service speed.

These humps are not important in large ships working at a speed of 10 or 11 knots, but are important in vessels of 300 to 500 feet in length running at 11 to 13 knots and above. Such forms should be tank tested, but for preliminary design purposes the following formulæ and table give particulars from which can be obtained the highest speed to which any form can be pushed without working on the bad part of the resistance curve. These highest speeds are given by :—

$$V=1{\cdot}45\sqrt{PL} \text{ for high-speed vessels.}$$

$$\left.\begin{array}{l} V=1{\cdot}05\sqrt{PL} \\ V=\ \cdot 85\sqrt{PL} \end{array}\right\} \text{for intermediates.}$$

It will be seen that the highest speed given agrees with that in § 27 when P equals ·53, a figure not unusual for vessels of this speed ratio. The following table gives the results for a number of vessels, all brought to a standard length of 400 feet, and are mainly based upon models tested in the William Froude tank.

TABLE 8.

Highest Economical Speeds for Vessels of a Standard Length of 400 feet.

Distinguishing Letter of Ship.	Prismatic Coefficient.	Length of Entrance in feet.	$\frac{V}{\sqrt{P L}}$
a . . .	·548	200	1·19
b . . .	·554	200	1·17
c . . .	·573	200	1·42
d . . .	·582	208	1·43
e . . .	·587	212	1·13
f . . .	·588	200	1·42
g . . .	·595	196	1·47
h . . .	·60	200	1·10
i . . .	·601	206	1·05
j . . .	·619	200	1·14
k . . .	·62	186	1·465
l . . .	·632	200	1·07
m . .	·639	160	1·0
n . . .	·655	179	·952
o . . .	·671	163	1·05
p . . .	·673	194	1·05
q . . .	·679	179	·86
r . . .	·684	195	1·07
s . . .	·686	200	·97
t . . .	·689	180	·935
u . . .	·70	188	·915
v . . .	·70	133	·91
w . .	·71	200	1·02
x . . .	·742	134	·805
y . . .	·745	152	·735
z . . .	·75	127	·72
2*a* . .	·758	134	·775
2*b* . .	·77	130	·75
2*c* . .	·775	134	·75
2*d* . .	·785	125	·705
2*e* . .	·799	89	·695
2*f*. . .	·816	100	·54
2*g* . .	·828	100	·59
2*h* . .	·828	100	·57
2*i*. . .	·828	100	·55
2*j*. . .	·85	100	·50
2*k* . .	·849	67	·595

NOTE.

(1) *a*, *b*, and *e* are vessels of very large beam, and for this reason, although they have comparatively fine forms, they are not suited for running at the highest speeds given by the 1·45 coefficient of the formula given on the previous page.

(2) 2*e* has a beam of only 42·7 feet, hence the small length of entrance.

(3) 2*k* has a very small beam, viz., 32 feet, the angle of entrance being the same as that of 2*e*.

(4) 2*g* has a straight line entrance, 2*i* has a slight hollow in the entrance, otherwise there is no difference between them.

(5) 2*j* is not a good vessel at any speed; it is too full.

Once a model of a ship has been tested, the importance and exact location of the humps are known, and it is not a hard matter to tell whether, at any speed, anything can be gained by change of dimensions or form. The amount to be gained or lost by such changes varies considerably, being about 3 to 6 per cent. for low-speed vessels, 8 to 20 per cent. for cargo-passenger vessels doing 15 knots on 400 feet length.

CHAPTER VIII

CURVE OF AREAS

§ 29.—It must be assumed that the designer has settled the displacement of the ship—the question of length has already been considered; and he is now confronted with the problem of the best way in which the displacement may be distributed in a longitudinal direction, *i.e.*, what type of curve of cross-sectional areas it is best to adopt in order that the resistance shall be kept as small as possible. A minimum beam is given by stability considerations, and the draft depends on questions of strength and depths of water in various channels; but one is here only concerned with the total area of the sections (which primarily depend upon the product of the beam and draft), and not the mode of distribution of the area over the section, which is dealt with in § 43. If for cargo-carrying reasons, or in order to get boilers or machinery in place, certain sections must be maintained of a certain shape and dimensions, the distribution of the area is partially settled, but one may assume, as is more generally the case, that there is considerable latitude as regards the sectional area at every point of the ship.

For vessels having no parallel body a curve of areas which in its characteristics is not unlike a versine curve with more or less snubbing at its ends is found to be very good for all speeds ranging about that given by :—

$$V = \sqrt{L}.$$

An example will make this clearer. In figure 14 *ABCBA* is a versine curve, *i.e.*, it has a prismatic coefficient of ·5 and at each end is tangential to the base. The points of inflection (BB_1) are situated at the mid-length of each body. Such a curve is too

fine, particularly at the ends, for any ordinary vessel, but this defect may easily be remedied by what is called snubbing.

Let it be assumed that the versine curve $ABCBA$ is to be snubbed 10 per cent. at the stern and 10 per cent. at the bow. Set out AF forward and AP aft such that $AF=\frac{AM}{10}$ and $AP=\frac{AM}{10}$, and with a mould draw in a new curve $PBCBF$ as shown. The form is now too short, but by increasing the horizontal ordinates in the ratio $\frac{AM}{AF}$ in the fore body and $\frac{AM}{PM}$ in the after body a new curve is obtained of the same length as before, and the

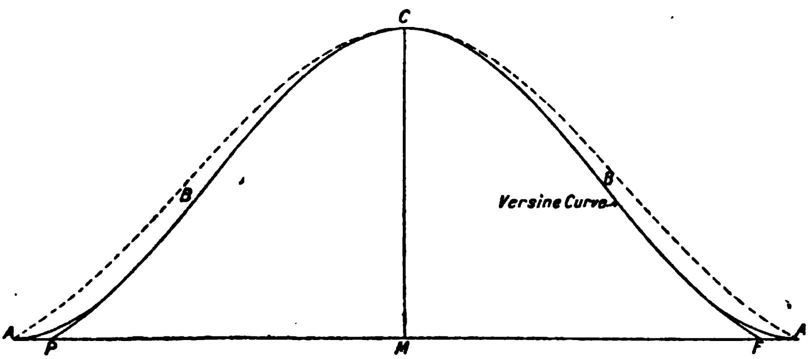


FIG. 14.

process by which this has been obtained is called "snubbing." The following table gives the increase in prismatic coefficient obtained by various degrees of snubbing of a versine curve, the effect of the snubbing being always confined to as small a length as possible consistent with a fair curve of areas. The effect of the snubbing upon the shape of the area curve is shown by the curves 2, 3, 4, 5 of Fig. 15 which are drawn with 5, 10, 15, and 18·16 per cent. of snubbing.

It will be seen that increase of prismatic coefficient obtained in this way is accompanied by a change in the character of the area curve, the hollowness of the ends becoming less marked with increase of snubbing, until it is eliminated altogether when the prismatic coefficient is ·6. This change in character of the curve

produces an increase of the resistance at all moderate speeds; but if the vessel is forced to speeds which are very high for its length,

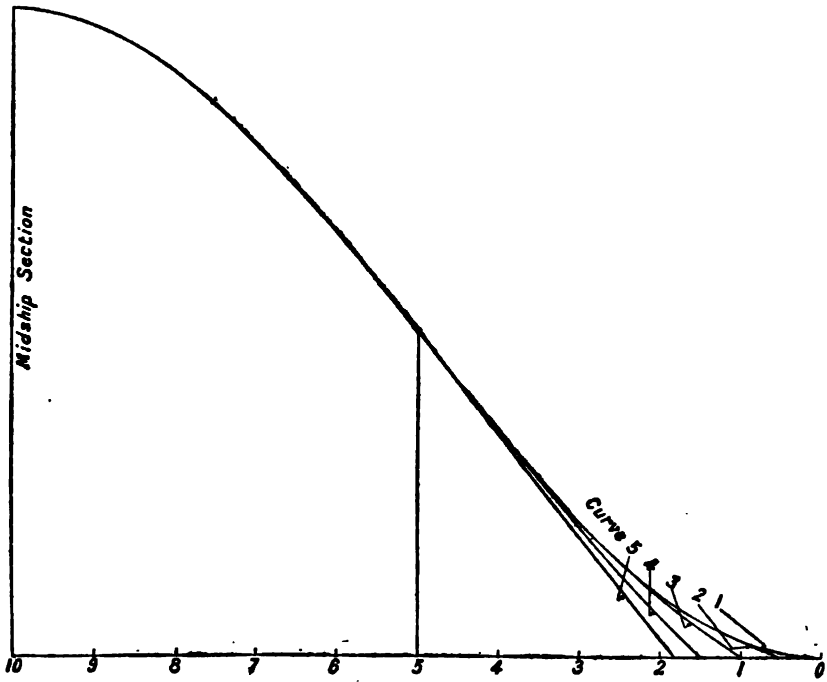


FIG. 15.—Curves of Area, derived from Versine Curve by "Snubbing."

then this area curve produced by the snubbing gives decidedly better results.

TABLE 9.

Curve.	Percentage of Snubbing.	Prismatic Coefficient of Snubbed Curve.
1	*nil*	·5
2	5	·526
3	10	·553
4	15	·58
5	18·16	·60

For prismatic coefficients above about ·55 the displacement can be worked into the ship either by having comparatively full ends

and no parallel body or by the introduction of more or less parallel body amidships between a fine but shorter entrance and run. It will be found that if two vessels of moderate speed have the same principal dimensions and displacement, one having a short length of parallel body combined with a fine and slightly hollow entrance, the other having no parallel body and therefore fuller ends, then the former will be slightly more resistful than the other at low speeds, and less resistful at the moderate speeds. The point at which the former becomes the better vessel and the measure of its advantage is difficult to settle other than by experiment, and we now proceed to examine the experimental data for fine vessels intended to run at moderate speeds.

§ 30. Low Prismatic Coefficient and no Parallel Body.—The most important experiments are those by Mr. Froude and Mr. Taylor. Mr. Froude's models were all of the cruiser type, *i.e.*, with ram bow and immersed counter. Six types of form were tried, and each type was tested at various M values or ratios of displacement to length. The principal coefficients of these are given in the following table, and the curves of areas and water lines for types 1, 3 and 6 are given in Fig. 16. The fore body shape is the same in types 1 to 3, the difference being in the stern, which was shortened by snubbing. This snubbing in fact had the double effect of making the stern both shorter and fuller. The after-body remained the same for types 4, 5, and 6, the bow being shortened by snubbing, the angle of entrance becoming greater the higher the type number.

It must be noted that the water line varied in a similar manner to the area curve, *i.e.*, it was hollow in the bow for types 1, 2 and 3, and gradually lost this hollow and became rounded in passing from type 3 to type 6.

Models of all the types were tried with ratios of length to breadth varying generally from 9·5 to 4·7.

The effect of form of area curve and water line (*i.e.*, of type) upon the general result was largely independent of the ratio of beam to draft, and only the results for the series in which

$\frac{\text{beam}}{\text{draft}}$ was $\frac{57}{22}$ are dealt with here. These are given in Fig. 17. Each curve in the diagram is for a particular (K) value, and gives the minimum (C) value obtained at that (K) value with any type of model, the best types being indicated by the numbers against the curves. As a rule (K) and (M) are known in the very earliest stages of the design, and these curves therefore enable the best type to be chosen for the design.

TABLE 10.

Form Coefficients for Froude's Models.

Midship Section Coefficient for all models . .	·8775
Ratio of Beam to Draft :	
First Series	$\frac{57}{22}$
Second Series	$\frac{66}{19}$

Type Number.	Prismatic Coefficient.		Remarks.
	Fore-body.	After-body.	
1	·54	·57	
2	·54	·6015	The cruiser stern was relatively larger on types 4 to 6 than on the others.
3	·54	·63	
4	·553	·63	
5	·5675	·63	
6	·60	·63	

The penalty for stern snubbing is never very great. Type 2 is better than type 1 for a fair range of speed, and type 3 never becomes more than 3½ per cent. worse than type 1 at any speed within the range of the experiments. For bow snubbing as represented by type 4 the excess resistance over that of type 3 is never large, as is shown by the table below, but type 5 compares

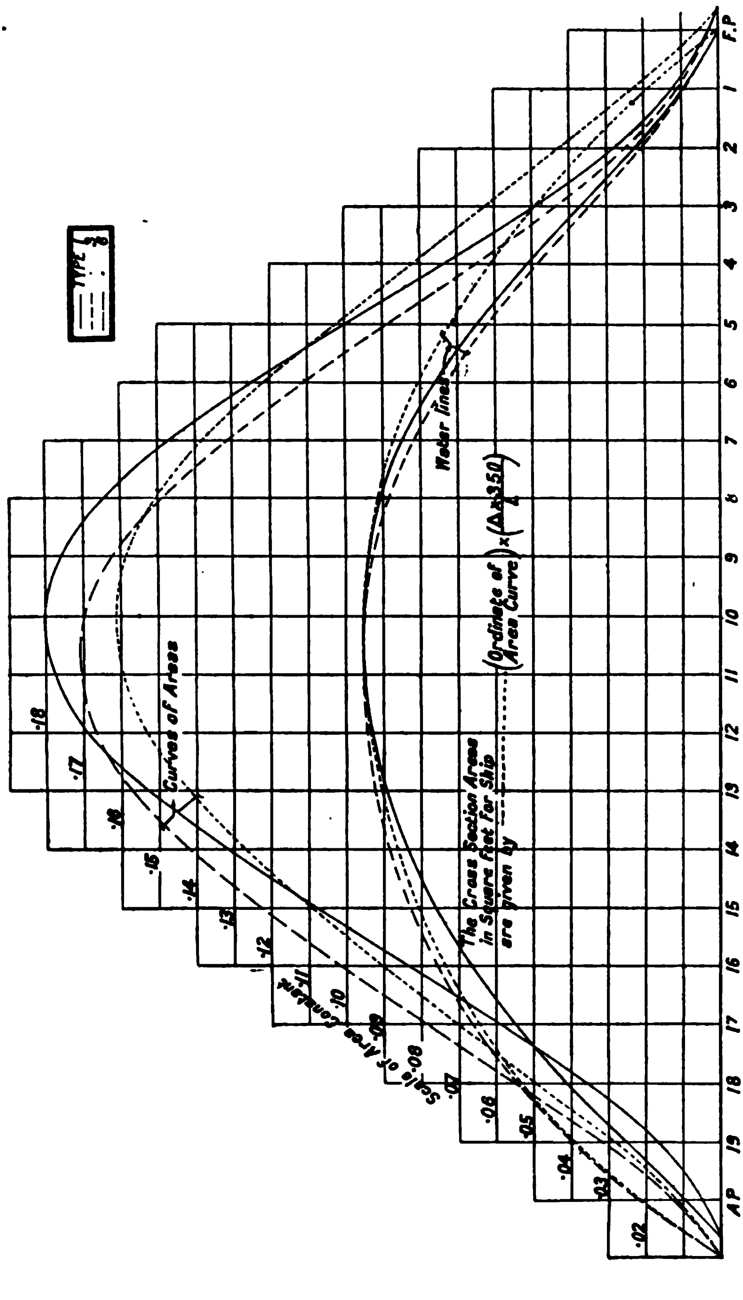


FIG. 16.—Curves of Cross Section Area and Water Line. R. E. Froude's Methodical Series.

The Area Curves are for Ships of the same length and displacement. The Water Lines are shown with the same beam, in order to bring out their differences.

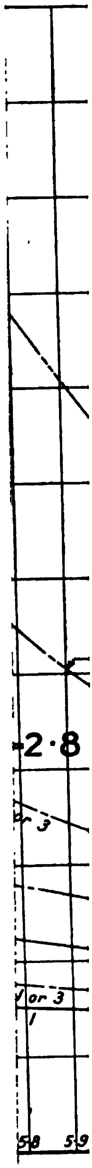


Fig. 1

very badly with the others at low speeds up to $(K)=2{\cdot}8$, when it begins to get better for the low (M) values, *i.e.*, for vessels of large beam for the length.

TABLE 11.

Increase in (C) passing from Type 3 to Type 4.

(K) Values.	2·0	2·2	2·4	2·6	2·8 and above.
Percentage increase of (C)	3·0	2·0	2·0	From 2·0 at $(M)=6{\cdot}5$ to *nil* at $(M)=4{\cdot}7$	Within 1 per cent. or better.

§ 31.—The experiments by Mr. Taylor with models having no parallel body were with forms having two prismatic coefficients, viz., ·60 and ·64, and with four types of area curve. Each type of area curve was tried with various water planes. These are shown for the ·60 prismatic coefficient in Fig. 18. Those for the ·64 coefficient were very similar, but fuller. They vary in character from the hollow-ended full-shouldered type, to the full-ended type with no hollow or even slight convexity in the area curve. The results may be summarised as follows :—

(*a*) The finer-ended forms have a very slightly larger skin area for given displacement. The change from area curve *A* to area curve *D* produces a decrease of 1½ per cent. in skin. This does not necessarily mean a decrease in resistance, as the increase of end fulness probably causes an increase of mean velocity of rubbing of the water against the form and thus increases the skin resistance per square foot of wetted surface.

(*b*) Assuming, however, that this decrease is actually realised,

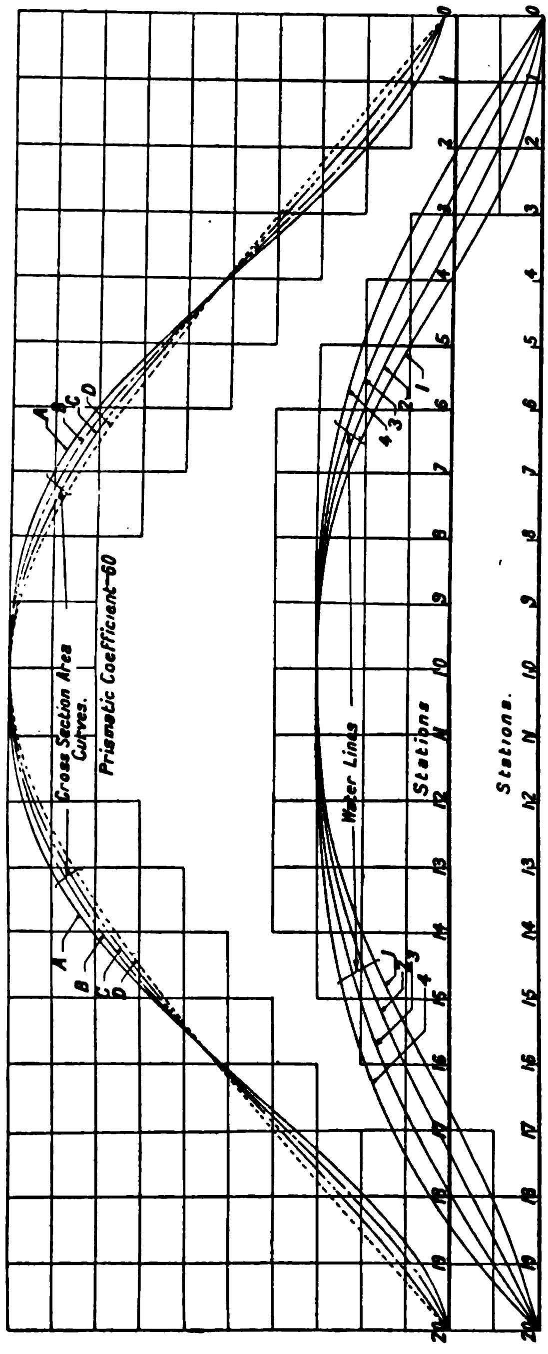


FIG. 18.—Curves of Cross Section Area and Water Line. Taylor's Models.

the hollow-ended area curve has lowest residuary resistance at all low speeds. When the speed exceeds that given by

$$V = 1{\cdot}1\sqrt{PL}$$

V being the speed in knots,
L the length in feet, and
P the prismatic coefficient,

the reverse is the case, and full ends and easy shoulders then show a considerable advantage, but the forms become rather wasteful at these higher speeds.

(*c*) For high speeds with the ·60 prismatic coefficient the best combination was area curve D having water line 1 forward and 4 aft. For speeds above $V = 1{\cdot}3\sqrt{PL}$ a finer water line aft gave a little better result than the above.

For the higher prismatic coefficient ·64 at high speeds there was little difference between the results with area curves C and D. For speeds above $V = 1{\cdot}12\sqrt{PL}$ the best result was obtained with either of these combined with a fine bow water line similar to 1 and a full water line aft similar to 4.

The full bow water line number 4 gave bad results with any of the area curves at high speeds.

(*d*) Varying the shape of the area curve in the after-body only, keeping the same water line, had little effect upon the residuary resistance, particularly at high speeds. At more moderate speeds lying between

$$V = 1{\cdot}1\sqrt{PL} \text{ and } V = {\cdot}7\sqrt{PL}$$

the fuller ended area curves showed slightly worse than the others.

§ 32.—The research work at the William Froude tank supports the above conclusions. For speeds ranging between $(P) = \frac{1}{\sqrt{2}}$ and $(P) = 1{\cdot}05$ (*i.e.*, for $V = {\cdot}75$ to $1{\cdot}1\sqrt{L}$ for a vessel having a prismatic coefficient of ·6) the following combination gives good results :—

The bow water line should be kept as fine as stability or cargo

considerations will permit, and should be drawn with a slight amount of hollow in it. The curve of areas for the fore-body should also have a fair amount of hollow in it and be somewhat of the character of curve 2 in Fig. 15. The curve of areas for the after-body should be much the same as that of the fore-body, but should be combined with a fairly full water line such as number 3 of Fig. 18. This gives good stability, easy buttocks, and comparatively open sections in the neighbourhood of the propellers, all of which are things to be desired. The full water line at the stern also prevents the ship from squatting abnormally when travelling at high speed.

If the water line at the stern is kept too full it results in blunt endings and consequent eddy-making, and partly to avoid this a "*cruiser stern*" has been adopted in many recent ships. For vessels whose speed is fairly high for their length this may be a real advantage. By carrying the cruiser stern formation well down the stern post the ship not only has a finer and more tapered water line, but the immersed form is given a good finish so far as curve of areas is concerned. Both of these are factors which go to the reduction of the residuary resistance, and both help to increase the speed to which the ship can be run economically.

For vessels of lower speed its main use is the avoidance of eddy-making, or the attainment of water line inertia, without the use of a large water-plane coefficient.

§ 33. Channel Steamers.—For vessels of the Channel steamer type, running at speeds above $V = 1 \cdot 2\sqrt{L}$, a form having practically no hollow in the bow lines is the best. The fine angle of entrance and gently sloping buttocks are still required. The maximum ordinate of the water line should be kept aft of amidships, and the maximum ordinate of the lower level lines may be brought forward of this point. A cruiser stern is of enormous value to these vessels, and if properly worked into the ship will give a reduction of 10 to 15 per cent. in power compared with a similar ship without such a stern. The main factor in these vessels

which is adverse to good results is large displacement. The more this can be reduced, *i.e.* the smaller the ratio of

$$\frac{(\text{displacement})^{\frac{1}{3}}}{\text{length}},$$

the better is the result obtained.

§ 34. High Prismatic Coefficient and Parallel Body.—From the results in § 29 it can be seen that to obtain a curve of areas having a prismatic coefficient greater than ·6 either something very like parallel body must be introduced into the form, or the area curve must be made convex along its whole length. A curve of areas which is wholly convex can be used with advantage in certain types of vessels. The after-body of destroyers, steam yachts, and other high-speed vessels is sometimes of this character, and in vessels of comparatively small draft in which gently sloping buttocks can be adopted such a type of area curve is not detrimental. But in the fore-body of a ship it is of advantage only at very high speeds, except for one class of ship, *i.e.*, the fullest type of large tramp steamer intended to travel at 9 or 10 knots. Each of these cases is dealt with later. For the more ordinary vessel the introduction of parallel body into the ship is more economical in the cost of building and in propulsion, and usually gives more stability.

This parallel body may be introduced into the ship in one of several ways :—

A. By fining the ends and filling out the shoulders so that the midship body becomes practically parallel for a certain portion of the length.

B. By increasing the ship's length by an amount equal to the length of parallel body added.

C. Adding the parallel body between the entrance and run and then reducing the whole length to its original value.

D. Proceeding as in case B, but afterwards reducing all the dimensions to bring the displacement to its original value.

Case A differs from the others, as the ends become finer as the parallel body is added. Thus the prismatic coefficient remains

constant with the change instead of increasing as it does in the other cases.

In *case B* the ship's length, displacement, and prismatic coefficient are all increased, and the ratio of beam to length is decreased.

In *case C* we have the same increase in prismatic coefficient as in B, but it is obtained with fuller angles in the lines of the entrance and run. The displacement added is of course less than in case B, being here proportional to the difference between the final and original prismatic coefficients.

In *case D* the form obtained is similar in every respect to that in case B, but the ship is smaller in all its dimensions.

The following table shows the effect of introducing 20 per cent. parallel body into a form whose dimensions and particulars are given in column O, the change being effected by the methods indicated above.

TABLE 12.

Column . . .	O	A	B	C	D
Variation by Method .	—	A	B	C	D
Length (feet) .	400	400	480	400	439
Beam (feet) .	50	50	50	50	45·7
Draft (feet) .	22	22	22	22	20·1
Prismatic coefficient . .	·65	·65	·71	·71	·71
Displacement (tons)	8,020	8,020	10,484	8,760	8,020

§ 35.—Mr. Taylor has made experiments with models whose forms have been varied by method A. Three forms having prismatic coefficients of ·68, ·74, and ·80 were chosen. All the models had a midship section coefficient of ·9, and the same ratio of beam to draft, viz., 2·5. Each prismatic coefficient was tried with five lengths of parallel body, the entrance and run being shortened and fined down as more parallel body was introduced. The change was made so that in every series, the form always had

exactly the same shape of transverse section at the point where the area was the same.

The area of wetted surface is increased very slightly as more parallel body is used, but the increase is negligible, as can be seen from the figures below.

The residuary resistance varied according to the speed and the fineness of the ends. At high speeds the most resistful were invariably those with the fine ends and most parallel body. At more moderate speeds there was a certain percentage of parallel body for each prismatic coefficient, which gave the minimum residuary resistance, this length varying a little with the speed.

TABLE 13.

		Prismatic Coefficient.		
		·68	·74	·80
Percentage increase of surface per 10 per cent. of parallel body used .		·35	·25	·15
Percentage of parallel body for minimum residuary resistance per ton at...... $\frac{V}{\sqrt{L}}=$	·5	8	22	31
	·6	12	27	35
	·7	13·5	27	34
	·8	12	24	31
	·9	4	18	26

At low speeds these percentages may be varied considerably, up or down, without much effect on the resistance. At higher speeds a 1 per cent. increase or decrease in parallel body causes a corresponding increase of roughly 1 per cent. in residuary resistance. It is fairly safe to assume that the frictional resistance per ton is constant. In this case, with a form whose residuary resistance is about 30 per cent. of the whole, there is a latitude of 3 per cent. of length, for which the penalty is 1 per cent. increase in resistance. This ratio of residuary to total resistance is a very fair average for a form suited to its speed.

The results were practically independent of the (M) value or ratio of displacement to length at any speed up to $\frac{V}{\sqrt{L}}=\cdot75$, which is as high as mercantile vessels of similar form would be pushed in practice. Broadly speaking, therefore, it can be said that, for the usual range of speed obtained by ships of these prismatic coefficients, parallel body may be used with advantage up to 12, 24, and 36 per cent. of the length, with prismatic coefficients of ·68, ·74, and ·80 respectively, and these percentages may be increased somewhat without any great loss.

The results obtained by Professor Sadler,* although not sufficiently extensive to define limits for the efficient use of parallel body, agree, as far as they go, with Taylor's work. A form having a prismatic coefficient ·87 with 80 per cent. of its length parallel body and a hollow-ended area curve, was considerably improved at all speeds by reducing the parallel body to 60 per cent. and straightening out the area curve. A second form of ·67 prismatic coefficient showed better results when the parallel body was increased from *nil* to 10 per cent. in the fore-body and the area curve at the bow given a little hollow. A similar change at the stern showed a slight increase in resistance. With a form having a medium prismatic coefficient of ·74, up to speeds given by $\frac{V}{\sqrt{L}}=\cdot5$, 20 per cent. of parallel body gave better results than either 40 per cent. with a hollow end or *nil* and a very full-ended area curve. Above this, and up to the practical limit of speed for such forms, the combination of hollow-ended bow and full-ended stern was best. It appears reasonable to suppose, therefore, that the after-body area curve can be changed considerably (consistent with fairness) without much effect, in forms whose prismatic coefficient is less than ·67. But with fuller forms, parallel body, although an advantage in the fore-body, should be used as little as possible in the after-body, as the

* American Society of N. A. and M. E., 1907 and 1908.

shorter run increases the resistance. This question of length of run is dealt with in the section on eddy-making and in §§ 41 and 42.

§ 36. Variation of Form by mode B (*i.e.*, by the insertion of Parallel Body between ends of identical Form).—Experiments were made by Mr. W. Froude with a series of models having a constant

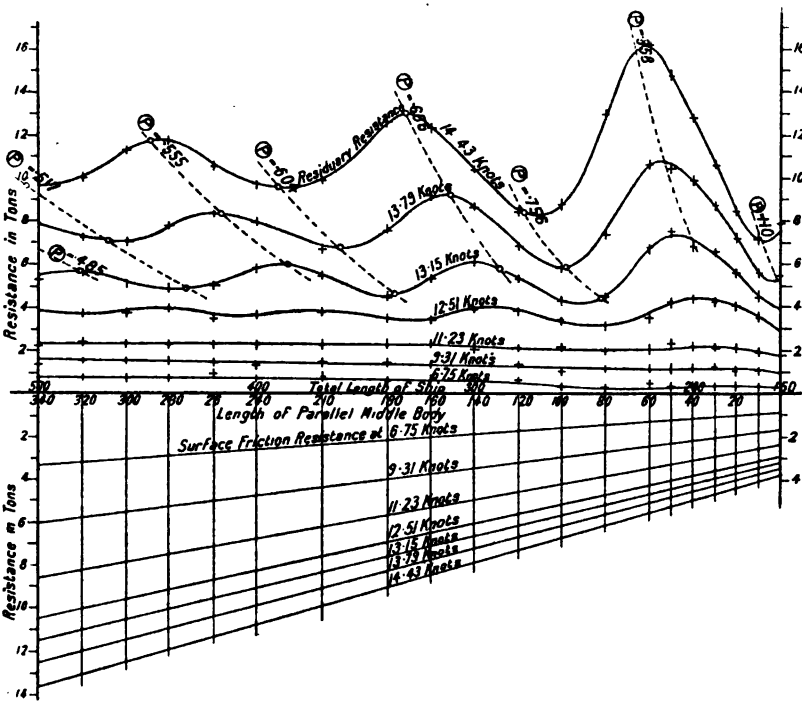


FIG. 19.—W. Froude's Experiments.

Displacement for 160 feet length = 1,246 tons. Each 10 feet of parallel middle body adds 142 tons to displacement.

ratio of beam to draft, viz., 2·67. The prismatic coefficient of the entrance was ·526 and of the run ·57. The largest section had a coefficient of ·9. The results are given for vessels having 38·4 feet beam, the length of entrance and run then being each 80 feet.

The resistance of the models was separated by Mr. W. Froude into frictional and residuary components (see Fig. 19). The former was assumed to increase uniformly with the length of

parallel body inserted, and when deducted from the whole gave the residuary resistance. For all speeds above 12 knots (for the ship dimensions given above) the latter was found to oscillate about a fairly steady value as the length was increased. The amplitude of the oscillation decreased with length, but became greater for higher speeds. Below 12 knots the residuary resistance increases steadily as the parallel body is increased. At 11 knots the increase is 10 per cent., and at 9·3 knots 25 per cent., for an increase of total length from 160 feet to 500 feet.

A little consideration of the figure will show that, from the point of view of power per ton of displacement, the vessel with long length of parallel body is generally better than a shorter vessel at all low speeds. This holds good for all his models up to speeds of about 12·0 knots, and for forms of length greater than 200 feet it applies up to 13·15 knots. This latter speed is, however, too high for reasonable propulsive efficiency, for which the upper limit is about 12·0 knots, and the experiments cease to have much practical value beyond this point.

But they have considerable scientific value in the fact that at the higher speeds they show clearly the effect of length upon the wave-making. Since the entrance and run remained the same, the oscillations in the resistance curves can be due only to the "phase interval" or distance apart of the bow and stern wave systems. The final system created is favourable or resistful, according to the relation of the speed to the ship's length. The hollows and crests of these curves all fall on curves of constant P, and the conclusion is that vessels intended to run at these high speeds should be designed for the P values of ·604 or ·756 or 1·10, according to the ship's length.

Similar experiments have recently been made at the William Froude National Tank, with fuller forms having the usual mercantile stern. The particulars of these, when enlarged to give the same length of entrance as before, are as follows:—

Length of entrance = length of run =	80	feet.
Beam	29·9	feet.

$\dfrac{\text{Beam}}{\text{Draft}}$	2·25 feet.
Prismatic coefficient of entrance . .	·672
" " " run . .	·638
Midship section coefficient . . .	·98

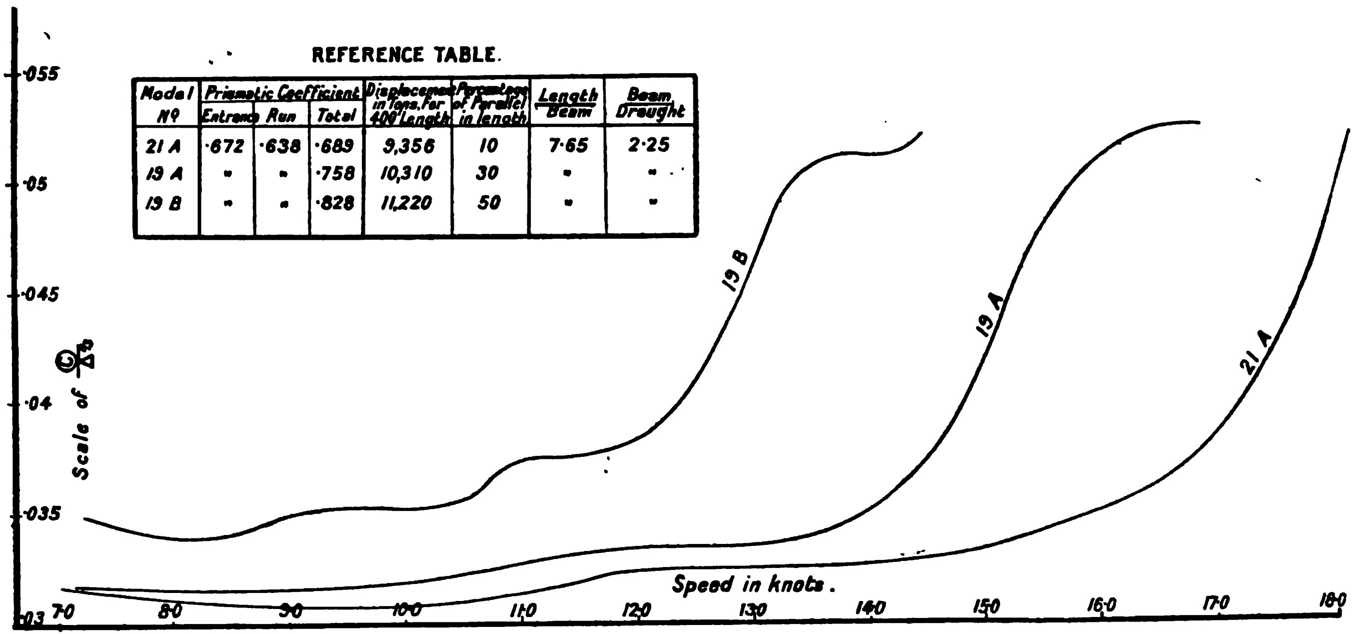
At low speeds, the power per ton became smaller the longer the length of parallel body introduced. At higher speeds the resistance curves showed the same oscillations as before, with their crests at the same (P) values. All the forms were wasteful for speeds above 11·5 knots (for the above ship dimensions), a speed slightly lower than that for Froude's model, owing to the fuller form of entrance.

Long narrow ships obtained in this way, although requiring lower power per ton than shorter ships of the same maximum section, do not necessarily represent the best that can be done on the displacement (see § 35 and § 45).

§ 37. Variation of Form by Mode C (*i.e.*, by making the entrance and run the same shape but of shorter length as parallel body is inserted between them, the total length remaining the same).—This has been tested in several cases at the William Froude tank. Fig. 20 shows the results for three models corrected for skin friction, so that the diagram is correct for ships of 400 feet length. All the forms had the same total length, breadth, depth and midship section, the prismatic coefficient of entrance being ·672 and of run ·638. For all the models the shape of the entrance is given by curves 8 in Fig. 21 and the run by curves 3 in the same figure. But as parallel body was introduced in passing from models 21*a* to 19*a* to 19*b* both the entrance and run became shorter, and in consequence the angle of entrance greater. The ordinates of the figure show

$$\frac{EHP}{\Delta V^3} \times 427 \cdot 1.$$

In other words, at any fixed speed given by the abscissa, the ordinates are proportional to the power per ton of displacement.



REFERENCE TABLE.

Model Nº	Prismatic Coefficient Entrance	Prismatic Coefficient Run	Prismatic Coefficient Total	Displacement in Tons, for 400' Length	Percentage of Parallel in length	$\frac{\text{Length}}{\text{Beam}}$	$\frac{\text{Beam}}{\text{Draught}}$
21 A	·672	·638	·689	9,356	10	7·65	2·25
19 A	„	„	·758	10,310	30	„	„
19 B	„	„	·828	11,220	50	„	„

Fig. 20.—Effect of Parallel Middle Body, with fixed Length.

The amount of parallel body that may be advantageously inserted depends entirely upon the speed of the ship. Thus below 13 knots the form with 10 per cent. of parallel body is but slightly better than that with 30 per cent, the difference being less than 3 per cent. But it is evident that to insert much more than 30 per cent. of parallel body would be detrimental, as the power per ton increases rapidly above this amount. The cause of this is partly the long length of body with sharp bilge turn, partly eddy-making at the stern, and partly that the entrance becomes shorter and unsuitable for the speed.

The above models had comparatively full ends. A similar comparison between models with 10 and 30 per cent. of parallel body with finer ends (having a little more hollow in the area curve forward) gave better results with the fuller form. The particulars of these models are given in Table 14.

TABLE 14.

Model.	Prismatic Coefficient.			Displacement in Tons for 400-foot Ship.	Percentage of Parallel Body in the Length.	$\frac{B}{D}$	$\frac{L}{B}$
	Entrance.	Run.	Total.				
14*b*	·57	·584	·621	8,450	10·45	2·25	8·0
18*a*	·57	·584	·687	9,570	30	2·25	8·0

Up to 12 knots for a 400-foot ship the fuller vessel requires 3 per cent. less power per ton displacement than the other. Above this speed the results were rather critical owing to a hump in the resistance curve, but even at a speed of $\cdot 8\sqrt{L}$ there was little difference between them.

Similar tests made with other models show that it is generally the case that a small percentage of parallel body may be added to a fine-ended ship without increasing the effective horse-power per ton for all moderate speeds. But if it be added to any great extent between ends which are already full, the form is only fit for quite low speeds, and even at these speeds the result may be

anything but satisfactory. Table 8 gives some values of the maximum economical speed for a number of forms, the models *2f*, *2g*, *2h*, *2i*, *2j* having 50 per cent. and *x*, *2a* and *2c* 30 per cent. parallel body.

§ **38. Variation of Form by Mode D** (*i.e.*, by adding parallel body between fixed entrance and run, and reducing all dimensions to bring the displacement to the original value).—The principal characteristic of this alteration is the reduction of the length of entrance and run. This limits the maximum speed for efficiency, which will vary approximately as $\sqrt{\text{length of entrance}}$. Consider the case of the vessels in the previous section, having an entrance length of 80 feet, and limiting speeds of 12·0 and 11·5 knots respectively. If, as a consequence of a modification of the form by the mode under discussion, the length of entrance is reduced to 60 feet, the new forms would be unsuitable for speeds above 10·4 and 9·1 knots. Other than this, the change does not have much effect. At low speeds there is a slight reduction of power per ton of displacement, which disappears as the above limiting speeds are reached.

CHAPTER IX

SHAPE AND FINENESS OF ENDS WITH PARALLEL BODY

§ 39.—In the section dealing with the relative merits of hollow *versus* straight lines, and elsewhere, it has been shown that for vessels of fine form intended to work at speeds in the neighbourhood of $V = \sqrt{L}$ there is a decided gain in working the level lines with some hollow in them. It has also been shown that for such fine forms at very high speeds the hollow should be reduced to get the best effect. The above conclusions, however, being based upon experiments with forms mainly without parallel middle body, do not necessarily apply to the majority of merchant vessels. It is the purpose of this section to consider what is the best prismatic coefficient of both entrance and run and best shape of area curve to associate with a given length of parallel body.

The experiments of Mr. Taylor, discussed in § 35, show that starting with fixed dimensions and displacement, fining down the ends more and more as parallel body is introduced amidships, a condition is reached at last, beyond which more parallel body and still finer ends would mean increased resistance per ton at a given speed. In these experiments increased length of parallel body was naturally associated with an entrance and run of increasingly hollow lines, which is not usually the case in practice and is not necessarily good in theory. The problem can, however, be viewed from a broader point of view if only the dimensions are kept constant and the displacement varied.

§ 40.—In order to test the effect of fine and full ends, a large

speeds. The ·672 entrance begins to get bad at (P)=·65, and a (P)=·7 it is 12 per cent. worse on (C), or 10 per cent. worse on E.H.P. per ton, than the form with ·625 entrance prismatic coefficient.

Put into everyday figures, two vessels each of 400 feet length travelling at a speed of 15·3 knots, one having a displacement of 9,400 tons with an entrance prismatic coefficient of ·67, the other having a displacement of 9,110 tons and an entrance prismatic coefficient of ·625, the former will require 12·3 per cent. more horse-power to carry only 3·5 per cent. more displacement. The difference between the full and medium entrance at the higher speeds is even greater than the above.

(b) *Varying Stern Prismatic Coefficient.*—Increase in fulness of the stern had practically no effect upon the horse-power per ton except at speeds between (P)=·65 and ·73. For many ships this is the usual speed range, and for these the finest stern is decidedly the best.

(c) *Varying Shape of Ends.*—Up to speeds of 11 knots for 400 feet length, there was no difference in power between the forms with varying hollow in the bow. But over a range of speeds from (P)=·6 to ·72 (*i.e.*, 13·3 to 16 knots for 400 feet length) the medium bow is better than either of the others. The straight line bow is very wasteful over this range, being from 6 to 10 per cent. worse than the medium bow.

Of the forms with varying amount of hollow in the stern, that with none at all gave slightly better results than the others up to about 15 knots for a 400-foot ship. The hollow stern then became the best, its advantage reaching a maximum of 4·5 per cent. and dropping to *nil* at (P)=·78.

§ 41. Medium Parallel Body.—In these models the parallel body was worked amidships for 30 per cent. of the length. The prismatic coefficients of the entrance and run were varied between

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the same limits and in the same manner, as in the set of models with 10·45 per cent. of parallel body. Where the entrance and run prismatic coefficients are given the same, they had the same curve of areas and load water lines (see Fig. 21), *only* the length being altered, so that the *total* length remained the same although the length of parallel body was increased.

The conclusions arrived at are as follows :—

(*a*) *Varying Entrance Prismatic Coefficient* (see Fig. 22).—At $P = \cdot 43$, or 10 knots for a 400-foot ship, the form with finest bow required 3 per cent. more power per ton than that with the fullest. But this disadvantage disappeared with increase in speed, and at $P = \cdot 53$ the power per ton was independent of the prismatic coefficient of the entrance (within the range of the experiments). Above this speed the finer entrance became necessary for good performance, and at the highest speed for which these forms are suited, $P = \cdot 59$, the finest form was 5·5 per cent. better than the medium and 11 per cent. better than the fullest entrance. The difference in displacement between the fullest and finest entrances was only 4·5 per cent. of the whole displacement, or only $\frac{1}{2 \cdot 8}$ the increase in power.

(*b*) *Varying Stern Prismatic Coefficient.*—Increasing the prismatic coefficient of the run from ·578 to ·638 had very little effect up to $P = \cdot 6$, the fuller stern showing slightly worse at higher speeds. But filling out the run to ·70 prismatic coefficient increased the power per ton for all speeds by an average of 4 per cent. This appears to be due to eddy-making, a conclusion which is supported by the experiments with the model having a stern of type 2 (Fig. 21). This had very hollow stern lines, which appear to be too sharp for the water to follow, and this model also had a C value 3 per cent. higher than others of the same prismatic coefficient but with straight lines. It would seem, therefore, that this

fullest stern, which has an after-body coefficient of ·79, is slightly over the border line of what is good for economical propulsion (see also § 35).

(*c*) *Varying Shape of Ends*.—The bow with the least hollow was the best for all speeds up to $\textcircled{P} = \cdot 52$, or roughly 12 knots for 400 feet length. But, just as with the series with short parallel body, the medium bow became decidedly better than either of the others over the moderate and more useful range of speeds, *i.e.*, $\textcircled{P} = \cdot 53$ to ·615, the advantage varying from 1·0 per cent. at both these speeds to a maximum of 7·0 per cent. between them.

Varying the shape of the run from the straight line (curve 4, Fig. 21) to the medium form had practically no effect except at speeds too high for economy, when the straight line form was the better. Extreme hollow in the stern produced eddy-making, as already mentioned.

§ 42. Long Parallel Body.—In these models the parallel body was worked amidships for 50 per cent. of the length. The same forms of run as before were tested with the medium entrance (·672 prismatic coefficient). The variation of the entrance with fixed run (·638 prismatic coefficient) was over a slightly larger range, viz., prismatic coefficients of ·764 to ·625 (see Fig. 21).

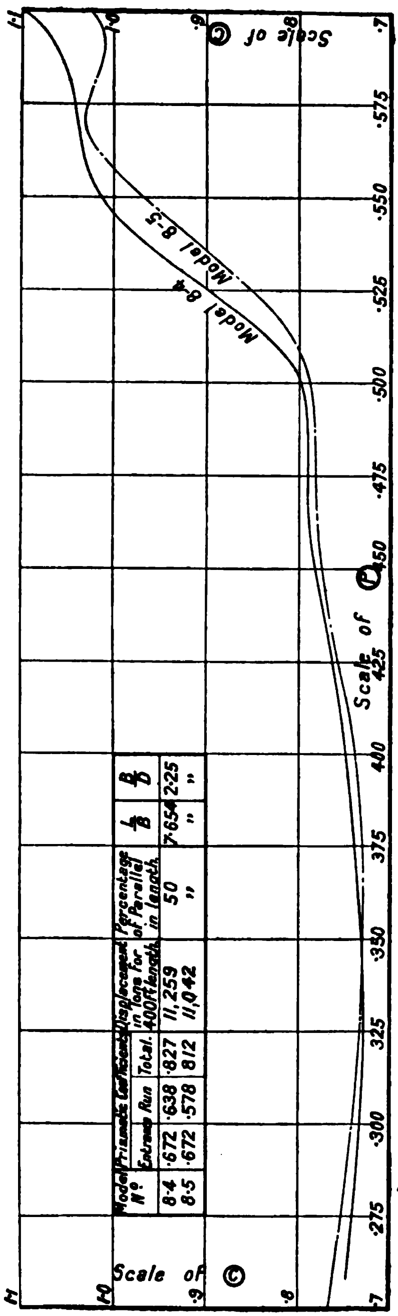
The general conclusions arrived at are as follow :—

Varying Entrance Prismatic Coefficient (see Fig. 22).—The power per ton remained the same for all the models up to speeds given by $\textcircled{P} = \cdot 375$. This corresponds to a speed of 9·1 knots for a vessel of 400 feet length. For higher speeds the full entrance became very wasteful. The medium form can be used for a little higher speed, but at $\textcircled{P} = \cdot 45$ and above it is 2·5 per cent. to 3 per cent. worse than the finest form. It will be noticed that these forms have not good entrances for speeds above $\textcircled{P} = \cdot 46$. The *length* of entrance must be increased and the parallel body in the

fore-body decreased if such speeds are to be reached with good Admiralty coefficients.

In this connection it is well to point out that vessels with excessively full bows and fine sterns are inclined to steer wildly and meet with greater resistance in rough water than a similar vessel with finer bow. For these reasons the author is inclined to think that, although these results show that a fore-body coefficient of ·88 can be carried at 9·1 knots (for 400 feet length) in smooth water, it is too full for ocean work, and should be reduced to about ·80 with a slightly lower proportion of parallel body than that adopted in the models being discussed. For higher speeds than (P)=·4 a still lower fore-body coefficient should be used.

Varying Shape of Entrance. — The form with straight line end and easy curvature at the shoulders (curve 9, Fig. 21) gave the best result. Its advantage



Model No.	Prismatic Coefficients Entrance	Run	Total	Displacement in tons for 400 ft. length	Percentage of Parallel in length	L/B	B/D
8·4	·672	·638	·827	11,259	50	7·654	2·25
8·5	·672	·578	·812	11,042	"	"	"

FIG. 23.—(C) Curves for two Forms with 50 per cent. Parallel Middle Body (see § 42).

is perhaps negligible at $\textcircled{P}=\cdot 35$ and below, but it increases to 3 per cent. and 5 per cent. at and above $\textcircled{P}=\cdot 45$. The hollow entrance gives a bad-looking model, and at all speeds requires 10 per cent. to 12 per cent. more power per ton displacement than either the straight line or medium entrance.

Varying Prismatic Coefficient and Shape of Stern.—In all the models with various shaped sterns eddy-making appeared to be present in a more or less marked degree. The two best models were those with (1) the finest after-body prismatic coefficient (·789 ; see curve 5, Fig. 21), and (2) the medium prismatic coefficient (·836) having straighter lines. The $\textcircled{C}$ curves for these two sterns associated with the same entrance (fore-body prismatic coefficient of ·836) are given in Fig. 23. In neither of them was the eddy-making very great, and the lower levels were quite clear of it.

If such high after-body coefficients are required, the curve of areas and water line should take forms similar to curves 4 and 5, Fig. 21, but with a somewhat easier shoulder for the latter curve. It must be remembered that one of the main factors in producing eddy-making in such forms as these is the ratio :—

$$\frac{(\text{area of midship section})^{\frac{1}{2}}}{\text{length of run}}$$

These model results show that in order that the stream lines shall not break away from the stern, the value of this ratio should not exceed $\frac{1}{4\cdot 08}$ (see also § 16).

CHAPTER X

POSITION OF MAXIMUM SECTION, AND RELATIVE LENGTH OF ENTRANCE AND RUN

§ 43.—The experiments of Professor Sadler with equal-ended models having prismatic coefficients of ·54 and ·61 and no parallel body showed that within practical limits the midship section position had little influence upon the resistance. If anything it appeared to be advantageous to keep it a little aft of mid-length. Similar and more complete experiments have been made at the William Froude tank with a large number of mercantile ship forms. In these experiments the total displacement, length of ship, length of parallel body in any set, midship section shape and area were all kept constant, but the parallel body was shifted to varying fore and aft positions relative to the perpendiculars, the entrance and run remaining the same in form, but being elongated or compressed as necessary. The parent forms tried are given in Table 15, p. 94, together with the range of ratio of $\dfrac{\text{length of entrance}}{\text{length of run}}$ covered by the experiments.

For the finest forms the results agree with those of Sadler, the best ratio of $\dfrac{\text{length of entrance}}{\text{length of run}}$ being 1·1 to 1·2, according to the speed, the latter becoming the better at really high speeds for the form. Beyond these limits the resistance increases continuously, particularly at wave-making speeds.

With series "B," which had a blunter bow than the fine series, the influence of the position of the parallel body was still small, and may be neglected for speeds below that given by a K value of 1·2 (or 9·3 knots for a 400-foot ship). Above this speed no

definite rule can be laid down. The experiments showed a very emphatic gain at some speeds by using a short entrance, but this advantage had disappeared at a $\textcircled{K}$ value of 2·2. This uncertainty is due to one of the wave humps, which depends upon the wave-making length of entrance. As the entrance is shortened and run lengthened, the wave-making, which is more noticeable owing to the blunter lines, occurs at lower speeds, and there is a waviness in the curve of resistance at fixed speed as the ratio of entrance to run changes.

TABLE 15.

Length of ship . . . . . . 400 feet
Midship section coefficient . . . ·98
Number of drafts in the beam . . 2·25

Set.	Beam in Feet.	Displacement in Tons.	Prismatic Coefficient.		Parallel Body in Feet.	Range of Ratio. $\frac{\text{Length of Entrance}}{\text{Length of Run}}$.
			Entrance.	Run.		
A	50	7,400	·52	·584	40	·76 to 1·67
B	52·26	8,450	·57	·584	41·8	·57 to 1·62
C	52·26	9,570	·57	·584	120	·55 to 1·68
D	52·26	10,810	·67	·64	120	·6 to 1·67
E	52·26	11,259	·67	·64	200	·6 to 1·65

The total prismatic coefficient of any form in the above table can be obtained by combining the entrance and run with the appropriate length of parallel body which has a prismatic value of unity.

The results with the forms having 30 per cent. parallel body combined with the finer ends (set "C") show that the ratio of $\frac{\text{length of entrance}}{\text{length of run}}$ may vary from ·9 to 1·2 without any material effect upon the resistance at any fixed speed below that corresponding to a $\textcircled{K}$ value of 1·6. As in any case it would not be economical to run a ship of this form at any speed above that given by $\textcircled{K}$ value equal to 2·1 (*i.e.*, a speed of 16·6 knots for a 400-foot ship), the results are fairly definite for this type.

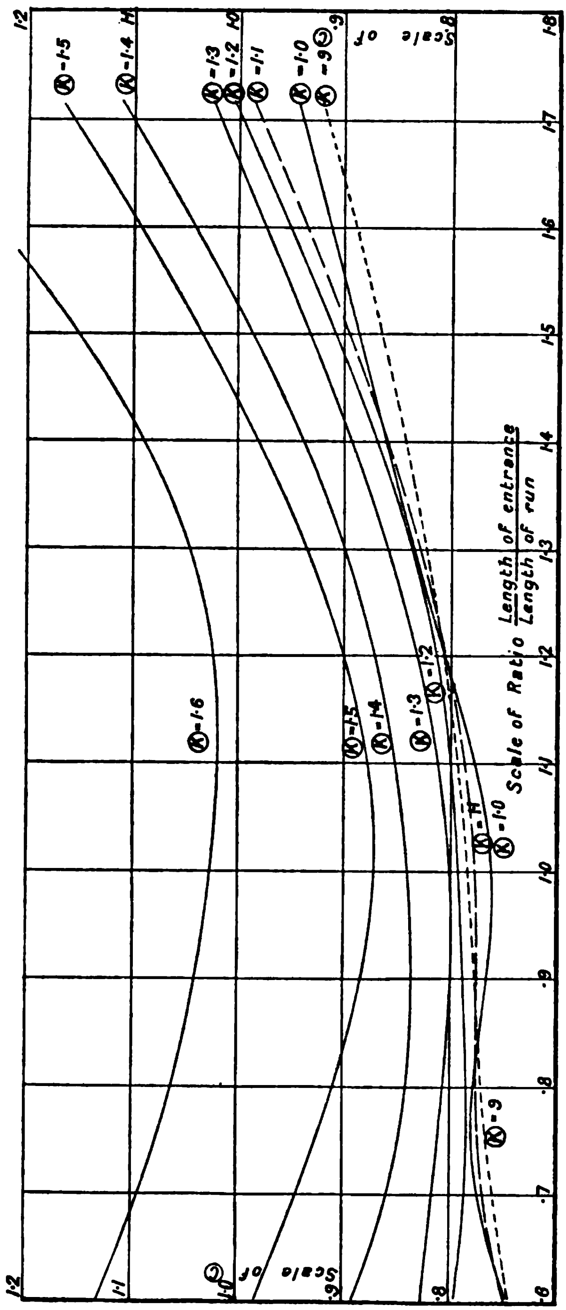


FIG. 24.—(C) Curves for Forms with 50 per cent. Parallel Middle Body.

Entrance as Curve 8, Run as Curve 5, Fig. 21.

With a fuller prismatic of both entrance and run, the forms with a short entrance and relatively long run are better than the reverse at all moderate speeds. In set "D" the advantage is but little, and if the speed exceeds that given by K equal to 1·5 the entrance and run should be of equal length. As the form becomes rather wasteful at K equal to 1·7 (or 13·6 knots for a 400-foot ship), the above gives, for all practical purposes, a fair indication of the effect of any such change in form.

But this gain with short entrance and long run becomes most marked when the parallel body amounts to 50 per cent. of the total length. The C curves for these forms are given in Fig. 24, and it will be seen that at low speeds the ratio $\frac{\text{length of entrance}}{\text{length of run}}$ should be about ·9 for the best results.

The explanation of this advantage obtained with short entrance and long run lies in the fact that as the parallel body is added the lines become blunter at the ends and a certain amount of eddy-making takes place. This can be at any rate partially eliminated by elongating the run at the expense of the entrance, and so long as the curtailment of the bow does not increase the wave-making more than is gained by the reduction of the eddy-making at the stern, so long is there something to be gained by the change. So far as could be told, the eddy-making begins when the parallel body amounts to about 30 per cent. of the total length. With a relatively smaller beam than that adopted in the experiments it is probable that a higher proportion of parallel body could be worked without any marked detrimental effect. Moreover, if the form has a very hollow curve of area at the stern it is more liable to this defect than one with practically no hollow at all. As it is of great advantage from the point of view of the efficiency of the propeller to do away with this eddy-making, vessels having a large proportion of parallel, or practically parallel, body in their length should have but the slightest amount of hollow in their curve of areas for the stern. (See also § 16 and § 42.)

CHAPTER XI

MIDSHIP SECTION AREA AND SHAPE

§ 44.—This is as a rule settled by considerations more important and quite apart from the question of resistance and propulsion, and model experiments show that within reasonable limits this practice is quite sound. The shape of the midship section can be varied over very wide limits without affecting the resistance provided the beam, water line, and curve of areas remain unaltered. With the full type of midship section commonly adopted in the merchant service a little qualification is required to this, as a too sharp turn of bilge has certain disadvantages which have been already pointed out; but good results can be obtained with midship section coefficients up to ·98 if the rise of floor is *nil* and a little less than this if the rise of floor is not abnormal. This general rule enables the designer to adopt any type of section which may suit the particular service of the vessel without having to trouble about the question of propulsion.

From the above it will be seen that the ratio of breadth to draft cannot be a very useful means of comparing different forms. A far better criterion is the ratio

$$\frac{\text{beam}}{\sqrt{\text{midship section area}}}=\frac{B}{\sqrt{A_m}}.$$

This ratio is independent of the form of midship section, and at the same time is a measure of the relative width and mean depth of the immersed form.

The question of absolute size of midship section is intimately connected with the one of longitudinal distribution of displacement, and from many points of view cannot be wholly separated from it. It is also closely associated with the stability of the ship,

since a large midship section coefficient means a comparatively low centre of buoyancy at the deeper drafts, and a large area of midship section with a moderate coefficient will give a higher centre of buoyancy and metacentre. The possible combinations are infinite, but the consideration of some more or less restricted cases will serve to show what is best for propulsion in most cases.

The usual type of merchant ship has from 7·5 to 8·5 beams in its length, and 1·9 to 2·8 drafts in the beam with a midship section coefficient of ·85 to ·98. But, supposing the beam and draft are severally or together increased, will this affect the resistance per ton ?

§ 45. Increase in Midship Section Area, Beam, and Draft without Increase in Total Displacement.—This necessarily involves a fining of the ends of the form and a reduction of the prismatic coefficient, which is favourable to the attainment of high speeds at economical rates. Mr. W. Froude has tested the effect of this on two forms of the following dimensions :—

TABLE 16.

Form.	Length in Feet.				Beam (feet).	Draft (feet).	Dis-place-ment (tons).	Wetted Surface (square feet).
	Entrance.	Parallel Body.	Run.	Total.				
A	144	72	144	360	37·2	16·25	3,980	18,860
B	179·5	—	179·5	359	45·9	18·0	3,980	19,130

Both had the same degree of fineness of entrance and run ; the larger midship section of form " B " was balanced by an extension of the entrance and run to amidships, the parallel middle body being entirely eliminated. The change from " A " to " B " involved an increase of wetted area, as indicated by the last column of the above table. Below 13 knots form " A " was very slightly the better, but above this speed form " B " improved continuously compared with " A " owing to the smaller wave-

making resistance consequent upon the decrease of the prismatic coefficient. Experience at the Froude tank shows that for vessels of fairly full form running at speeds above or about that given by $V = \cdot 68\sqrt{L}$, within reasonable limits, greater beam and finer ends will give better results.

§ 46. Increasing Beam and Draft at every Section, keeping the Midship Section Coefficient and Form of Level Lines unaltered.— Such a change as this leaves the prismatic coefficient unaltered, but the displacement and angle of entrance of the water lines are both increased. The vessel with the greater beam has the smaller area of wetted surface per ton displacement, which has to be balanced against the increased wave-making due to the larger angle of entrance.

Froude's methodical series of model experiments show that with models of prismatic coefficients ·55 to ·61 the broader the vessel the smaller is the power per ton at all low speeds. Thus at $K = 2{\cdot}4$ the power per ton decreases as M decreases (or beam increases), the reduction for 10 per cent. in displacement (*i.e.*, 5 per cent. increase of both beam and draft) being about 1 per cent. This advantage is a very small one, and as speed increases the increased importance of the wave-making causes the broad vessel to slowly lose in comparison, and it becomes the worst at high speeds. The change over takes place at $M = 5{\cdot}9$ to $7{\cdot}4$, according to the speed, but above $K = 3{\cdot}3$ the vessel with the smallest beam is always the best.

Taylor's experiments show much the same result. His models had prismatic coefficients of ·56 and ·68, the former being fine ended, and the latter, although having no parallel middle body, being rather flat amidships and fairly full at both ends. At low speeds the effect was the same as is given above for $K = 2{\cdot}4$. At high speeds, given by $V = 1{\cdot}1\sqrt{L}$, with the ·56 prismatic coefficient the beam change had very little effect on the power per ton, but with the ·68 prismatic coefficient the broadest vessel was the worst.

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§ 47. Decreasing the Midship Section Coefficient by increasing the Beam and Draft together, the Area of each Section remaining the same.—Taylor has made experiments with models varied in this way. Two sets were tested having prismatic coefficients of ·56 and ·68 respectively. Several ratios of displacement to length, covering all the practical range, were tried. The midship section coefficient was varied from 1·10 to ·70, corresponding to $\frac{B}{\sqrt{A_m}}$ values of 1·63 and 2·04. The ratio of beam to draft was maintained the same for all the models, viz., 2·923. The forms with fine midship section coefficient naturally had "peg-top" sections, and those with large midship section coefficient had an underwater bulge amidships.

The minimum wetted surface for the models was obtained with midship section coefficients of ·9 to ·98 for the ·56 prismatic coefficient and ·86 to ·93 for the ·68 prismatic coefficient.

With both prismatic coefficients a change of $\frac{B}{\sqrt{A_m}}$ from 1·63 to about 1·76 (midship section coefficients 1·1 to ·95) had no effect upon the power per ton.

For the finer midship sections not only was the wetted surface greater, but the residuary resistance increased as beam and draft were increased, at all speeds up to the maximum consistent with economy of propulsion (V equal to 1·48 and 1·15 times $\sqrt{PL}$ for the fine and full prismatic coefficient respectively). The excess of the total resistance of the ·7 over the ·95 midship section coefficient amounted to approximately 7·5 per cent. for the finer vessels and slightly less than this for the full vessels at their normal speeds. At lower speeds the difference was not quite so great in either case.

The results show that for the attainment of high speed there is nothing to be gained by the adoption of high values of $\frac{B}{\sqrt{A_m}}$ in order to get a fine midship section.

§ 48. Varying $\frac{\text{Breadth}}{\sqrt{A_m}}$ by increasing Beam and decreasing Draft,

keeping Displacement, Form of Sections, and Levels unaltered.—It can quite easily be seen that a large increase of beam on a fixed displacement and length will generally mean an increase of wetted surface, and generally it can be stated that with given form of section there is a particular value of ratio of $\frac{B}{\sqrt{A_m}}$ which has the advantage of offering a minimum wetted surface, which for low-speed vessels is a very important item.

In Colonel Rota's experiments with models having a midship section coefficient of ·87, and prismatic coefficient of ·56, this best ratio of $\frac{B}{\sqrt{A_m}}$ is about 1·85, and there is remarkably little change of wetted surface between values of 1·6 and 2·3. So far as frictional resistance is concerned, therefore, these figures give the best proportions of $\frac{B}{\sqrt{A_m}}$ for this form of boat. But the experiments showed that the residuary resistance increased with the beam at all speeds, and as a result the minimum value of *total* resistance for a vessel of 328 feet length occurs at a somewhat lower value of $\frac{B}{\sqrt{A_m}}$ than that for minimum wetted surface, viz., 1·38 to 1·8. The extent to which these latter ratios are lower than the ratio for minimum wetted surface must depend upon the form of the ship—*i.e.*, its curve of areas, etc.—but for good forms there can be little doubt that the above result is fairly representative.

Departure from this best value of $\frac{B}{\sqrt{A_m}}$ in the direction of smaller beams was not a matter of very great moment for the range of variation possible in most cases. But the resistance increased fairly uniformly with this ratio when it exceeded 1·8, and when it became 2·45 the resistance was 13·5 per cent. above the minimum. This can be seen from Fig. 25, which gives the C values for ship of 328 feet length, to a base of ratio of beam to draft. The forms with the largest beam and smallest draft may

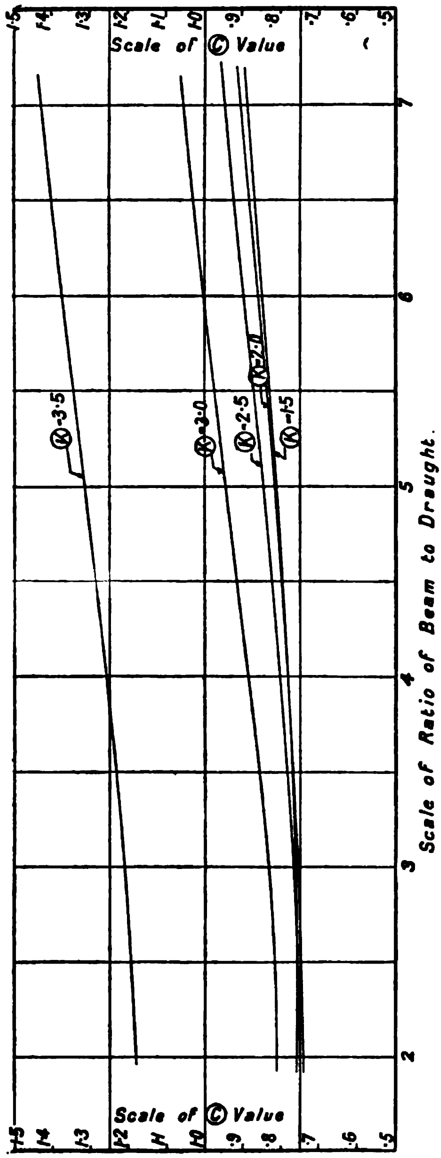


FIG. 25.—(C) Curves. Rota's Experiments (see § 48).

be classed as *shallow-draft vessels* of high speed, as the draft corresponding to 328 feet length is only 9⅓ feet. If the midship section coefficient be increased from ·87 to ·97 the draft would

become 8·36 feet and the results would still apply within 1 per cent.

Mr. Froude's experiments show that a change of $\frac{B}{\sqrt{A_m}}$ from 1·72 to 1·99, obtained in the same way, caused an increase of resistance of approximately 3 per cent. for vessels of high (M) value or small ratio of displacement to length, and 6 or 8 per cent. for vessels of larger and more usual ratios of displacement to length. The models with which the above experiments were made are described in § 30 and Table 10.

§ 49. Decreasing Midship Section Area by increasing the Prismatic Coefficient, all Dimensions and Displacement remaining the same.—Taylor has tested this with models having block coefficients from ·56 to ·68. The models had a ratio of beam to draft of 2·4, and the dimensions were arranged so that the product

$$\text{beam} \times \text{draft} \times \text{block coefficient}$$

was the same in all the models. For any block coefficient the curve of areas had more hollow the larger the midship section coefficient; and the area curve for the largest block coefficient (·68) had nearly straight ends at ·98 midship section coefficient and bluff-rounded ends for smaller midship section coefficients. The water lines followed the area curves in shape. The sections had a slightly bulbous stem, and both stem and sternpost were vertical.

The variation of wetted surface is very small with such changes, being of the order of 1·6 per cent. when the midship section coefficient is varied from ·86 to ·96 and 3·1 per cent. going from ·96 to 1·06 at all block coefficients.

Generally speaking the results show that at all moderate speeds a full midship section is good, and a coefficient of ·98 is quite safe even for a block coefficient of ·56. Except at very high speeds for the form, ·96 to ·98 may be taken as good working figures. At low speeds the full midship section gave slightly higher resistance. The speeds at which it became advantageous to reduce the area of midship section and fill out the ends of the ship are indicated

in the table by the heavy lines. Each line is for a definite $\frac{V}{\sqrt{L}}$ value, and for any block coefficient, that prismatic coefficient is best which lies immediately to the right of the line corresponding to the speed at which the vessel is to run.

TABLE 17.

Dimensions and Coefficients of Models.
Length . . . 20 feet.

Block Coefficient.	Breadth (feet).	Draft (feet).	Midship Section Coefficients.					
			·86	·92	·98	1·04	1·10	
			Prismatic Coefficients corresponding to above Midship Section Coefficients.					
·56	2·928	1·22	·651	·609	·571	·538	·509	
·60	2·828	1·179	·698	·652	·612	·577	·545	$\frac{V}{\sqrt{L}}$ = ·88
·64	2·789	1·141	·744	·696	·658	·615	·582	„ = ·72
·68	2·657	1·107	·791	·739	·694	·654	·618	„ = ·6
								„ = ·48

Thus for a block coefficient of ·68 at speeds between $\frac{V}{\sqrt{L}}$ equal to ·48 and ·6 the best prismatic coefficient is ·694 to ·654, corresponding to midship section coefficients of ·98 and 1·04 respectively. It must be noted, first, that the possible gain in resistance was always small, and, secondly, that the large midship section coefficients were obtained by large under-water bulging of the form, *i.e.*, the tumble home commenced well under water, and there were never any sharp or angular corners in the sections at the bilge.

§ 50. Varying $\frac{B}{\sqrt{A_m}}$ by Bodily Sinkage of the Ship.—The follow-

ing table gives the percentage increase in power for a 10 per cent. variation of displacement obtained in this way :—

TABLE 18.

Variation of Effective Power for 10 per cent. Variation in Displacement.

Type of Ship.	Block Coeffi-cient.	Length (feet).	Speed in Knots.						
			10	15	18	20	22	30	40
Battleship .	·65	400	—	7·0	8·0	12·0			
Cruiser .	·58	400	—	6·6	7·8	—	11·0		
Small cruiser	·51	400	—	8·0	9·0	9·5	11·2		
Destroyer .	·5	400	—	5·6	—	8·1	—	10·8	10·6
Steam yacht.	·5	400	—	6·5	7·8	—	10·1		
Passenger steamer .	·59	400	6·2	7·7	8·5	12·4			

			Speed in Knots.				
			8	10	12	14	16
Intermediate steamer .	·677	400	5·0	5·5	6·8	6·9	19·0 to deep line 12·0 to light line
Slow steamer	·758	400	4·5	6·5	6·6	7·0	10·0 at 15 knots

It will be noticed that the effect for each form varies with speed, and for all forms at low speeds the effective horse-power per ton of displacement decreases as the draft increases, and may be taken as varying with (displacement)$^{\frac{2}{3}}$. At those speeds at which wave-making is important the power per ton is roughly constant for all normal variations of draft.

The skin friction effect would be the same at high or low speeds, and the increased percentages in the table at high speeds must therefore be due to increase in wave-making. At the low speeds this depends upon the form near the water plane, but at higher speeds upon the cross-sectional area, which increases with the

draft. Larger waves, therefore, are created at the same speed as the draft is increased. For this reason, such a variation of power per ton of displacement as is shown for the "intermediate steamer" at its high speed is found, to a greater or less degree, in all ships, when they are forced beyond their proper speed.

CHAPTER XII

LEVEL LINES AND BODY PLAN SECTIONS

§ 51.—Assuming that a satisfactory curve of areas has been decided upon, and that the over-all dimensions are fixed, it remains to draw in the body plan sections and level lines. The most important is the normal level line. This being fixed, the sections may be drawn in almost any manner consistent with fairness and the curve of areas, without any material effect upon the resistance.

If the water line coefficient in either body is the same as the prismatic coefficient, the sections tend to become bulbous in character—*i.e.*, broader below the water line than on it—and for this reason the water line coefficient is usually made a little greater than the prismatic coefficient of the body. A slight increase in fulness of the water line enables the bilge curve to be eased, and avoids this tendency to bulbous sections and gives a much better bow line. Still more increase in area of water line will give sections decreasing continuously in width towards the keel.

Vessels of Low Speed.—For vessels having prismatic coefficients of about ·7 to ·8 intended for low speeds given by P values up to ·5 (or about 11·5 knots for a vessel of 400 feet in length) a comparatively full-ended bow water line can be worked without any material disadvantage. It gives bow lines with a good fore and aft slope and easier stream line flow under the form. For this class of vessel care is required in working the lower level lines into the parallel body, especially if the midship section is fairly full. A midship section coefficient of about ·98 with a little rise of floor

to the section gives a very quick turn at the bilge, and in a form having more than about 25 per cent. of parallel body is liable to show a corner where this sharp turn is merged into the tapering entrance and run. This can be avoided by adopting a flat floor line amidships and a bilge curve of larger radius, keeping the midship section coefficient the same; or of course this coefficient can be reduced, but the latter method involves making the ends fuller to compensate for the reduction of area amidships. It can also be avoided by easing the lower level lines so that the perfectly parallel body does not extend so far either forward or aft at the bottom as it does at the load level. This gives a good shape, and is to be recommended for all vessels of large prismatic coefficient. The bow lines become easier and the flow of water to the screw is better.

§ 52. Vessels of Higher Speed.—For these the case is different. In the full vessels easy stream lines are important, but with finer prismatic coefficient assuming a fair form easy stream lines are assured, and the avoidance of wave-making will now require special attention.

TABLE 19.

Type.	Ratio $\frac{\text{Beam at Half-length}}{\text{Beam Amidships}}$	
	Entrance.	Run.
Battleship . . .	·67	·88
First-class cruiser . .	·67	·88
Third-class cruiser .	·67	·82
Destroyer . . .	·68	·98

Skin friction does not enter into the question at all. With a fixed curve of areas the effect of any change of water line coefficient upon the area of wetted surface is very small. With ordinary forms having U-shaped sections in the fore-body, and V or Y-shaped sections in the after-body, the water line coefficient may

be varied widely and the skin area will remain practically constant. But variation of the area of water plane affects the stability, and a comparatively large water plane is usually necessary to secure the required stability in the ship. As a rule it is obtained by filling out the after water line. The amount of filling can be roughly gauged from the figures in Table 19, p. 108, which gives average values of the ratio of beam at the half length of entrance and run to beam amidships for several types of ship :—

This filling out of the after water line gives the sections a V or Y shape, according to the relative fulness of after-body and water line. It also has the advantage for all vessels of high or moderate speed that it reduces squatting, and in twin screw vessels the full stern protects the propellers when the ship is going alongside wharves, etc. Taylor's experiments with models having the same dimensions and fore-body, and the same curve of areas for the after-body, showed that the *shape* of the section had no very large effect on the resistance. The models had a prismatic coefficient of ·62, and a midship section coefficient of ·98. The best result was obtained with an after-body in which all the sections were geometrically similar to the midship section, *i.e.*, had vertical sides and flat floors, beam and draft being both decreased as necessary to obtain the desired area. This was some 3 per cent. to 5 per cent. better than the conventional form of stern. The worst shape was one in which the full beam and a flat floor were retained in all the sections, this being some 7 per cent. to 10 per cent. worse.

So far as resistance is concerned a comparatively full water line aft is slightly better than a fine one for all moderate speeds (see Fig. 26). This requires a little qualification in the case of certain single-screw ships. In such vessels having a screw with the blade tips 2 or 3 feet below the normal load line aft, the water line should be kept fine enough to avoid any dead water, and the body plan sections can be filled out towards the keel without material detriment to the resistance or the screw action.

§ 53.—So much freedom of treatment of the bow level lines is not possible if good results are to be obtained at moderate or high speeds. A fine entrance is essential for small wave-making. For vessels having 10 to 25 per cent. of parallel middle body a fine entrance with a little hollow in the water line gives better results at the service speed of such vessels than a full entrance with straighter lines, even though in the latter case the entrance is longer than in the former. The essential thing is not so much length of entrance as fine angles, which the former secures. For finer vessels both length and fineness of entrance are required to attain high speeds.

Fig. 26 gives the residuary resistance for four of the models tested in connection with the experiments detailed in § 31. These four models have the same area curve. Numbers 1 and 4 have a fine water line in the stern with fine and full bow water lines respectively. Numbers 3 and 2 have a full water line in the stern with fine and full bow water lines respectively. These water lines are given in Fig. 18. It will be seen that the full bow water line gives double the residuary resistance obtained with the fine and hollow water line.

For *high-speed vessels*, *i.e.*, those whose normal speed V is greater than $\cdot 9\sqrt{L}$, it is more difficult to lay down a general law. The best form of water line depends to a certain extent upon the shape of the adopted curve of areas. Taylor's experiments show that with a full-ended curve of areas such as C and D, Fig. 18, at all speeds it is an advantage to keep some hollow in the bow water line. But fine forms, whose area curves are hollow at the ends, as curves A and B, Fig. 18, give best results with only a moderate amount of hollow in the water line, and for speeds above that given by $V=1\cdot 1\sqrt{L}$ the form with the straight bow water line is the best. At these high speeds the form was slightly improved by fining down the stern load water line from its usual full form to a straight or even hollow curve. It must be remembered that the above results were obtained with forms whose prismatic coefficients were ·60 and ·64 having no actual parallel body. The possible reduction of power by variation of the load-

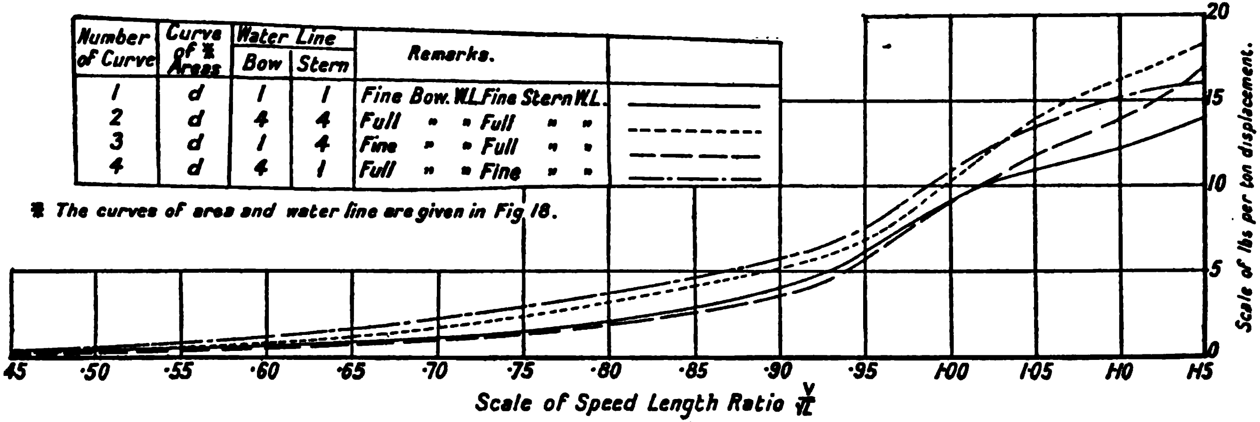


FIG. 26.—Effect of varying Bow Load Water Line.

Curves of Residuary Resistance in lbs. per ton Displacement.

line in these experiments amounts to about 6 per cent. of the whole for the practical range of variation.

§ 54. Hollow *versus* Straight Lines.—This question has already been considered for slow and intermediate steamers, and this section deals with the case of high-speed liners, cruisers, etc. Vessels of still higher speed, such as destroyers, are considered in § 55. The half-breadth lines in the fore-body of the majority of fast liners, cruisers, and battleships are hollow towards the end. The same remark applies to the lower level lines in the after-body. With low prismatic coefficients hollow level lines are a necessity if a curve of areas suitable for high speeds (*i.e.*, with hollow ends) is adopted, and the fore-foot and dead-wood aft are not cut away. It is possible, as in the case of sailing yachts, to maintain such a curve of areas and yet work the level lines without any hollow. But the block coefficient of such vessels is very low, and the sweeping contours of stem and stern post adopted in them are possible to only a limited extent in large vessels.

For fine forms, the experiments by Mr. R. E. Froude, made specially to test the relative merits of hollow and straight-ended forms, give reliable data. His models had the usual cruiser type of stern, and for one set of experiments corresponded to the following dimensions for ship :—

TABLE 20.

Form.	Length between Perpendiculars (feet).	Beam (feet).	Draft (feet).	Displacement (tons).	Midship Section Coefficient.	Prismatic Coefficient	
						Fore-body.	After-body.
D	490	74·5	26·0	14,458	·926	·552	·598
E	490	75·5	25·0	14,458	·90	·580	·639

Both models had the same displacement and length, and the same height of metacentre, the beam being varied in order to obtain this. In model "E" the level lines were practically

straight towards the ends in both bodies. The lines of model "D" had the usual amount of hollowness for such forms. As "E" had a greater prismatic coefficient than "D," its draft was reduced in order to maintain the same displacement. The tests showed "E" required 8 per cent. more power to obtain the same speed. This large difference extended down to quite moderate speeds given by $V = \cdot 8\sqrt{L}$. Tests made with other models of the same type showed that the major part of this effect was produced by the change in the fore-body.

A still greater difference was obtained with two models having the same principal dimensions and midship section. One had perfectly straight ends to all the level lines, and the other had the usual amount of hollow. The increase in displacement was about 7 per cent. (4 per cent. in the bow and 3 per cent in the stern), and the increase of power 13 per cent. at moderate and 16·5 per cent. at high speeds.

With regard to the effect on propulsion of the straightening out of the after-body lines, Mr. Froude's conclusion is: "The net upshot may be said to have been purely neutral as between straight and hollow lines for after-body, the balance of efficiency advantage which the screw experiments attributed to the hollow-line after-body being just about cancelled by the shaft tube and web advantage of the straight line one."

This increase in resistance with the straight level lines is due to a great extent to the fact that the curve of areas obtained by their use is a bad one for all moderate and high speeds. For very high speeds it may be, and is in certain cases, an advantage.

Experiments with these models were also made in what represented a very steep ground swell, all the waves having the same period. The increase in power required to maintain the speed was about the same with both straight and hollow line models, and was always greatest when the period of the waves as encountered by the ship, was some 12 per cent. greater than the natural pitching period of the ship. Although not conclusive, the results give no reason to suppose that with ordinary waves the smooth water advantage of the hollow line form is ever lost.

CHAPTER XIII

RACING AND OTHER HIGH-SPEED VESSELS

§ 55.—This term includes all such vessels as pinnaces, destroyers, motor launches and hydroplanes, *i.e.*, vessels whose highest speeds exceed that defined by the formula

$$(P) = 1{\cdot}5, \text{ or } V = 2\sqrt{PL}.$$

Thus for a vessel of 325 feet length having a prismatic coefficient of ·6 this critical speed is 28 knots.

At the above speed the hollows and crests of the bow transverse wave system are coincident with those which the stern tends to form, and as a consequence wave-making is abnormal. But above this speed the bow system gets out of phase with the stern system, and as the speed is increased each tends to cancel the other and the height of the transverse waves formed at the stern of the vessel decreases. But the divergent waves emanating from the bow also tend to increase in size, and unless the vessel is specially shaped to prevent it, this increase is continuous. So long as the cancellation of the transverse waves mentioned above exceeds the natural growth in the divergent wave system, the performance of the vessel will improve. This is what happens with many destroyers whose lines are very fine. The divergent waves formed are comparatively shallow and have but a small velocity in the forward direction.

Fig. 27 is a typical (C) curve for a good type of destroyer, and it will be seen that above $(P) = 1{\cdot}5$ the (C) value steadily decreases, but is tending to the horizontal at the highest speeds.

With such vessels the percentage of the propulsive power required to overcome the wave-making has been found to be

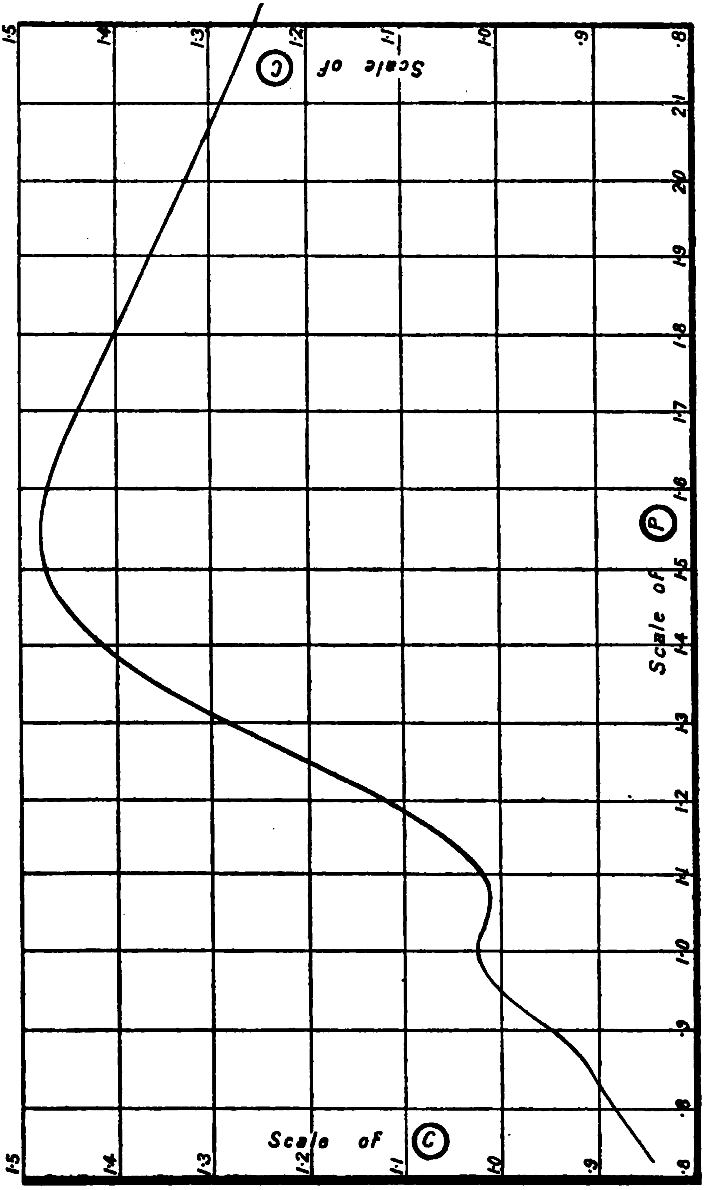


FIG. 27.—Typical (C) Curve for "Destroyer" Form.

the same, and in some cases to be less at 50 knots than it is at 30 knots. The longer the water plane of the entrance and the gentler the curvature of the buttocks the less important the

divergent waves become, and these two things therefore favour the attainment of very high speeds.

The curve of areas should have its maximum ordinate somewhat aft of amidships. It should have only a slight amount of hollow in it forward, and none is necessary aft. The shape of the curve of areas is not, however, of paramount importance, and may be varied within considerable limits, provided that in the operation due and proper regard is paid to fairness of the curve, and that the length of water line entrance is not materially curtailed. This elasticity of the area curve applies particularly to the after-body where the buttock lines are of most importance.

Good water feed to the propeller is the essential thing in the after-body, and this can only be obtained at high speeds with flat buttocks. These flat buttocks can be worked with quite a full water line, so that a fairly full-ended curve of areas can be obtained in the after-body. The load water line in the fore-body should not have any trace of hollow in it, and should have its maximum ordinate as far aft the midship section as possible. The maximum ordinate of the lower levels can be brought forward of the midship section without any detriment to the resistance, and this helps the buttocks a little.

The introduction of a slight amount of parallel body would not have much effect at the very highest speeds, but in the neighbourhood of $P = 1 \cdot 5$, and particularly for speeds slightly in excess of this, parallel body would be a distinct disadvantage, as bad interference of the bow and stern systems of waves would begin earlier and last longer. As parallel body in a fixed length also means a slight increase of angle of entrance, it is liable to produce worse divergent waves, and therefore higher resistance even at more moderate speeds, than the above.

One other factor which is of greater importance than the area curve is the M value or ratio $\frac{(\text{displacement})^{\frac{1}{3}}}{\text{length}}$. The higher this ratio, generally speaking, the higher is the hump in the C curve

and the greater is the power per ton to reach any speed in the neighbourhood of, or exceeding, the critical speed given on the previous page. To increase the length, means a greater weight of hull, and up to the point at which the gain in machinery weight due to the decrease in power required, exceeds this increase in hull weight, something is to be gained by increase of length.

§ 56. The Racing Motor Boat and Hydroplanes.—The highest development of the destroyer form is the racing motor boat. The hydroplane is the natural descendant of the latter, and the floats fitted to hydro-aeroplanes are simply hydroplanes built to satisfy certain special stability requirements.

Of the energy put into the water in the entrance in the form of wave or stream line motion, at the highest speeds, when V is about $2{\cdot}5$ to $4{\cdot}0\sqrt{L}$ very little if any can be recovered at the stern, and the object to be aimed at in such a case is to reduce the water disturbance to a minimum. The disturbance created in the water by a travelling area of pressure depends upon two things—the intensity of the force, and the length of time it is acting at any point. The former depends upon the angle through which the stream lines are deflected by the passage of the boat, and upon its speed. The latter varies inversely with the speed. All parts which meet the water should therefore have the easiest possible angles and smallest possible curvature on those lines along which the water will naturally flow.

In a racing motor boat the stream line flow is mostly along the bow and buttock lines, and these are more important than the levels. The comparatively flat buttocks required in a destroyer become still more necessary here. The attempt to lift the buttocks at the stern to any considerable extent would simply mean the formation of a suction area and consequent dead water under the stern, with large trim and increase of resistance.

Since these vessels lift their bows only partially out of the water, the long entrance is still required. The maximum ordinate of the normal still water plane may be placed well towards or even at the stern, as this gives easy angles, but no hollow is required in

TABLE 21.

Racing Motor Boat Data.

Name.	L feet.	$\frac{L}{B}$	$\frac{B}{D}$	Coefficients. Midship Section.	Coefficients. Prismatic.	Δ tons.	Half-angle of Entrance at L.W.L.	Greatest Breadth of L.W.L. from stem.	Remarks.	Speed V attained (knots).	P	$\frac{\Delta^{\frac{2}{3}} V^3}{\text{B.H.P.}}$
Napier I.	39·9	7·98	7·94	·95	·55	1·8	8°	·57 × L	Very flat sections aft	18·88	3·34	151
Legree Hotchkiss	39·3	7·87	4·55	·83	·45	2·26	8·5°	·63 × L	Flat sections aft	26·58	4·70	231
Hutton II.	39·9	8·5	4·95	·77	·64	2·53	8·4°	·67 × L	Round to V sections forward, flat sections aft.	25·1	3·71	209
Maple Leaf II.	40·0	4·44	—	—	·62	6·45	—	—	Step 19·5 feet from stem, knuckle line in sections forward, flat sections aft.	49·5	7·4	600
Air machine float.	13·3	5·6	2·23	1·0	—	·446	—	·55 × L	Step at ·55L from stem mudguard to step.	17·0 lowest planing speed.		
A hydroplane	20·0	4·0	—	1·0	—	·72	—	—	Viper type with tunnel forward, square sections aft, semi-submerged propellers.	22·0	—	341
Irene	39·7	6·4	6·9	·67	·61	2·5	9·0°	·70 × L	V sections forward and amidships to flat ones aft.	25·7	3·9	157
Crocco Ricaldoni boat.	26·25	—	—	—	—	1·475	—	—	Supported on V planes forward and aft.	37·8	—	690
Miranda III.	22·5	3·22	—	·95	assumed ·62	1·15	—	—	No step to bottom	27·0	about 5·4	394

this level, and it may even be rounded slightly, as this gives flatter bow lines.

In order to reduce the resistance due to the air and rough water, the bow should have a certain amount of flam with a turtle back. When the sections are formed with a knuckle line or chine, this should be kept a little above the water forward so that it remains above any small waves, as the trim is always somewhat less in rough water than in smooth. Table 21 gives the best available data for such vessels. It will be noticed that these motor boats have a beam of 5 feet or above, this being found necessary for stability under helm with the high power motors.

In all ships there is an upward resultant force at the bows due to the downward deflection of the water. Compared with the displacement of the ship, this dynamic force is quite small in the ordinary steamer, but it becomes of increasing relative magnitude and extends over a greater length of the ship as speed is increased, and if the hull is formed to take advantage of this force, the vessel will lift partially out of water. In a true planing condition the vessel is wholly supported by the impact of the water on its bottom, which is sufficient to balance the weight of the vessel and any suction that may exist at the stern.

Experimental data for such boats is very scarce. It appears that the effective horse-power of a hydroplane increases up to a certain speed when the plane rises to the surface of the water. As speed further increases, the power first drops, then gradually increases—approximately as the speed. Experiments made with floats for hydro-aeroplanes show that the value of $\frac{\text{displacement}}{\text{resistance}}$ is about 3·8 to 4·5 at the critical speed when the plane begins to lift to the surface. This ratio increases to about 7 when the hydroplane is planing on the surface at the top speed. These ratios depend to a certain extent upon the angle of inclination of the bottom, the above being approximately correct for angles between 4 and 6 degrees. Experiments with a flat plate and with models, show that the most efficient angle for the bottom is from

$3\frac{1}{2}$ to $5\frac{1}{2}$ degrees at all speeds; and most hydroplanes run between these limits.

The following are some general conclusions based upon experiments made at the Froude tank and upon the close examination of results attained in actual practice :—

(*a*) The keel forward of the step should be straight with very little rise at the stem, and the sections should have considerable flam at the fore end. This form can be brought to the surface with less power than is required for the toboggan shape, *i.e.*, that having parallel sides and shaped contour of bottom.

(*b*) The transverse sections of the bottom must be flat or nearly so, particularly in the after-body. A convex section is stronger than a flat one, but its lifting power is less.

(*c*) Air must be admitted to the step, if there is one. This can best be done, either by breaking the chine line at the step, and giving it less width aft of this point, or by a number of air pipes to the step. The former is much the more effective method.

(*d*) A hydroplane which runs partially on the tip of the stern will do so with less water disturbance if this tip is a horizontal line rather than a point, and in plan the stern of the boat should be very blunt.

(*e*) The speed at which any boat will plane depends largely upon the ratio of the displacement to the effective lifting area of the bottom. Thus, if

Δ is the displacement in pounds,
A is the area of bearing surface in square feet reckoned to the aftermost step,
V is the velocity in feet per second at which planing is to commence,

this ratio is given roughly by

$$\frac{\Delta}{A} = \cdot 12V^2.$$

Experimental results differ somewhat, but the above applies fairly well for models with surfaces inclined at 3 to 5 degrees. Higher values of this coefficient ranging up to $\cdot 15$ have apparently

been obtained in practice, but owing to the meagre data usually published, and to its ambiguity, a certain amount of caution is required in accepting them at their face value.

Seaplane or aeroplane floats are required to satisfy much the same conditions as hydroplanes so far as propulsion is concerned, but in addition, have to meet some special stability requirements. To prevent the seaplane from overturning when accidentally settling at a small angle (*i.e.*, down by the head) the floats must extend well forward of the centre of gravity of the machine and have good breadth to them. The step, if one is fitted, should be on a vertical line slightly aft of the centre of gravity of the machine when at its planing angle, *i.e.*, with the bottom of the floats inclined at about 4 degrees to the horizontal. This ensures the tail of the machine lifting clear of the water at a relatively low speed, and if the machine settles with a large angle the water force on impact tends to restore it to its correct attitude. It involves at high speeds the introduction of a moment tending to keep the bow of the machine down, but the air balance of the wings and fins can be arranged to counterbalance this without any help from the elevating planes.

Transverse sections of the floats should have flat bottoms just forward of the step. Since a broad flat surface cannot be made strong without heavy stiffeners, narrow floats are generally preferred. These narrow floats with good overhang forward can be given very easy buttocks and are generally satisfactory. For carrying a machine of 2,200 lbs. total weight on two floats the over-all dimensions of each would be approximately as follows :—

Narrow parallel sided type :

Length, 14·2 feet ; breadth, 2·3 feet ; total depth, 1·4 feet.

Flam bow type :

Length, 14·2 feet ; breadth, 2·4 feet ; total depth, 1·47 feet.

If the air wings will support the machine at a speed of 35 knots, the water resistance reaches a maximum at a speed of about 16·5 knots, and for a machine of the above weight, the *effective* power to overcome the float resistance is then about 22, if the

float has a good shape. At higher speeds the power for the water resistance rapidly diminishes owing to the decreasing load taken by the floats. Twin floats may be placed any distance apart which leaves a water gap between them of at least 3 feet. With less than this, over a certain speed range, the disturbance of the water between them grows, and with a very narrow water gap, a narrow ridge of water is thrown up at the centre to a height of several feet.

CHAPTER XIV

APPENDAGES

§ 57. Shaft Casing and Bossing.—In deciding upon the system of propulsion the effect of shaft casing and bossing upon the resistance must be taken into account. The shafts and bossing are bound to cause resistance, and this must be reckoned as a definite set-off against any other advantages accruing from increasing the number of propellers. Sub-division of the power leads to greater security against total disablement of the propeller apparatus, and facilitates sub-division of the ship. Twin-screw ships are handier in manœuvring than single-screw ships, and owing to the smaller diameter of their propellers, the screws are immersed better at the lighter drafts or when pitching. Against these advantages must be set a possible slight increase in engine-room staff and in first cost of machinery. But from the propulsive point of view the following points have to be considered :—

(1) With twin screws it is possible to obtain a larger area of screw surface on a given draught.

(2) The screws are situated in a better position relative to the stream line action, and better efficiency can be obtained.

(3) Higher revolutions can be adopted with twin than with single screws—an advantage in engine design.

(4) With every outboard shaft there is a certain loss due to the wash of the water past the brackets and shafting, etc.

The first item affects the backing and manœuvring power, as well as the thrust per unit area of surface. The second and third are outside the limits of this section.

§ 58. Resistance of Shaft Casing, etc.—Shaft casing and bossing

are required to facilitate examination of the shafting, etc., but so far as resistance is concerned serve only to prevent eddy-making where the shafts leave the ship. For this they should be kept short and small; where there is an acute angle between web and ship's side a "pocket" should be avoided by filling out the root of the web, and the after-edge of the web should have a taper finish of 1 in 4. The fashioning and web should be worked so that it causes minimum interference with the general flow of the water past the ship, which is shown by Fig. 5.

The shaft itself must necessarily meet water which is flowing obliquely across it, and eddy-making must take place to a certain extent. A casing and web tends to prevent this eddy-making, but against this must be set the additional skin friction of the web.

The resistance of shaft struts depends in a large measure upon their cross-section. With the usual blunt fore-edge and tapered after-end, provided this taper is not greater than 15 degrees, eddy-making due to form is avoided, and increase in length beyond what is required for this causes increase in resistance simply because of the increase in wetted area. The following formula gives approximately the resistance of struts provided that they are not put across the stream of water passing them :—

$$\text{Resistance in lbs.} = \cdot 044\, A \times V^{1 \cdot 83}.$$

A being the area of the two sides of the strut,

V the velocity in knots through the wake water.

On an average this amounts to about 2 per cent. of the whole resistance of a slow-running twin-screw ship.

The most complete experiments with shaft bossings are those by Mr. W. J. Luke. These were made with a model of a twin-screw ship having a ratio of breadth to length of 6·8 and a block coefficient of ·65. The webs and bossing were set to various angles from the horizontal, and experiments were made with and without screw propellers behind them. The diameter of the casing was ¾ inch, the length of the model being 200 inches. The spread of the screw centres was 5 inches to middle line, equal to one-sixth the breadth of the model.

It was found that the resistance of the model varied considerably with angle of bossing, as shown by the following table.

TABLE 22.

Effect of Angle of Bossing on Resistance of Model.

Angle of Bossing to Horizontal.	0°	22½°	45°	67½°
Resistance with bossing and webs compared with naked model resistance . .	1·097	1·04	1·026	1·05

The experiments of Mr. R. E. Froude and Professor Sadler show the same advantage attaching to the bossing inclined at 45 degrees, and experiments with a model of the *Kaiser Wilhelm der Grosse* show the same increase in resistance due to fitting horizontal casings and webs.

If the effect of the webs upon the action of the propellers be taken into account (as it should), Luke's experiments show that the direction of rotation of the screws is important, and an angle of 45 degrees, although about right for inward turning screws, is too much for outward turning.

Exactly what is best for the latter is not known. The high wake produced at the propeller by the horizontal webs is not good for efficiency, particularly with fast-running engines, and, although the hull efficiency is increased by it, this was found by Mr. Froude to about balance the loss due to fitting the webs horizontally rather than at 45 degrees. The possible loss in propeller efficiency with the horizontal webs, makes it doubtful whether there is any net gain by fitting them, and, as it is fairly certain that they are more resistful than the inclined webs, it seems better in the present state of our knowledge to work, even with outward turning propellers, webs inclined at an angle of at least 22½ degrees but not exceeding 45 degrees.

§ 59. Rudder and Bilge Keels.—The resistance of an ordinary unbalanced rudder, if properly tapered at its after-edge so that

no eddy-making takes place, can be estimated from its total area, regarding the resistance as being solely due to skin friction. The frictional coefficient should be taken as that given in Table 1 for the last foot of the 50-foot plank, with varnish surface, the formula becoming :—

$$\text{Resistance in lbs.} = \cdot 0087\, A \times V^{1\cdot 83}.$$

A being the wetted surface (both sides) in square feet,
V the speed through the wake water in knots.

For many forms this resistance may be neglected, as the rudder area is small and the forward velocity of the wake water is very considerable. This is particularly the case in vessels with full after-lines, as incipient eddy-making is very probably present near the rudder-post. A model having a prismatic coefficient of run equal to ·652 and length of run equal to 2·95 beams, showed no difference in resistance whether the rudder was fitted or not.

If the rudder is under-hung, the frictional coefficient must be taken for its own length, and if it is placed immediately behind the propeller, the velocity should be taken as :—

$$(\text{velocity of ship})\,(1+s)\,(1-w),$$

where

s is the slip of the propeller,
w is the wake coefficient.

The bilge keels, if properly placed—*i.e.*, with their planes following the general stream flow past the ship—can be treated in the same way as the rudder. Another and simpler method is to reckon the area of the bilge keels as additional wetted surface of the ship and to increase the resistance due to the skin in the same proportion. If the bilge keels are placed obliquely to the flow of the water, the resistance will be increased very considerably, the amount varying with the sharpness of the keel edge and the angle of obliquity (see § 17). This eddy-making resistance can, however, be entirely avoided with care, without any detriment either to the efficiency of the keels to prevent rolling or to their use as docking keels.

CHAPTER XV

RESTRICTED WATER CHANNELS

§ 60.—If a vessel is passing through a channel of gradually restricted cross-sectional area, the cross-section of the stream tubes around the ship must also be gradually reduced, and the changes of velocities and pressures must therefore be increased. An example showing this effect for a given form has been worked out. Fig. 2 shows the pressures along the stream form in a channel having an area equal to six times the sectional area of the form, as well as in an infinite fluid. It will be seen that not only are the pressure changes increased by the smallness of the channel, but the distribution is altered, there being a longer belt of reduced pressure or increased relative velocity.

Since the frictional resistance of any surface varies approximately as the square of the velocity of the water passing it, and this is increased in shallow water, there will be an increase in total resistance of any ship under these conditions. The percentage increase of *frictional* resistance will be practically the same at all speeds, since it only depends on the increase of stream velocity, which is independent of speed and dependent only on the relative cross-section areas of channel and vessel, or in shallow water with no side boundaries on the ratio of the mean depth of the vessel and the depth of water.

In shoal water the conditions become more favourable for eddy-making. Owing to the greater obliquity of the stream lines there is a larger demand upon each particle for a greater rate of change of pressure and stream line velocity, if it is to follow the ship's form at the stern, and therefore a greater chance of the stream line flow breaking down. The effect is somewhat the

same as with water flowing from a small into a larger channel. This also is an effect which is present at all speeds, and if it could be said that the formation of eddies was entirely independent of speed (which is roughly although not quite true), this effect also would entail a constant percentage increase of resistance at all speeds.

As the waves formed by a ship are due to the pressure disturbances caused by its movement, it follows from what has been said, that there will be a tendency toward greater wave-making in shallow water, provided that the waves created are related in speed, length, etc., in the same manner as with deep-water waves. But shallow-water waves, irrespective of their origin, differ in several respects from deep-sea waves. The crests are more peaked, and the hollows are more extended, the wave therefore being of that changed character which the change in stream line pressure round a ship in shallow water tends to produce.

Professor Havelock has shown that a travelling point of pressure creates waves of greater divergence in shallow than in deep water, and this divergence increases with speed, and ultimately at a critical speed they are all concentrated in a large transverse wave somewhat similar to a wave of translation. Above this speed only divergent waves are possible. The critical speed is given by

$$V^2 = 11{\cdot}5 \times d,$$

V being the speed in knots, d the depth of water in feet. It is reasonable to suppose that if this pressure were distributed about the supposed travelling point its effect would be much the same, and in a ship the formation of a large transverse wave may be expected at this critical speed. This is actually what takes place. As the critical speed is reached the transverse waves become more marked and the vessel trims rapidly by the stern, due to the pressure changes in the water. At the critical speed the bow is lifted on a long shallow bow wave, and a single broad transverse wave is formed at the stern. The latter wave is steeper on its front face than on its rear, and in some cases gives

the impression that it is about to poop the vessel which is creating it. At higher speeds the wave pattern consists of a well-marked, divergent system with only a slight indication of transverse waves, and the vessel trims somewhat less by the stern than in deep water. The extent to which this phenomenon increases the wave-making resistance, depends upon the relation of the critical speed for the depth of water, to the wave-making speeds of the vessel. The velocity of a wave of *given length* in shallow water is less than that of equal length in deep water. It follows that a ship will tend to make waves at a lower speed in shallow than in deep water. The more nearly a ship's natural wave-making speed approaches that critical speed appropriate to the depth of water she may be running in, the greater and more emphatic will the wave-making become.

There is a fairly complete and reliable collection of data on this subject, which may be divided into two classes, that pertaining to vessels whose speed is :—

(1) High for their length, such as destroyers, motor boats, etc.;

(2) Moderate for their length.

§ 61. High-Speed Vessels.—Fig. 28 shows the results obtained in shallow and deep water with H.M.S. *Cossack*. The dimensions of the ship and the water depths are given on the diagram. It will be seen that the speed for maximum increase of resistance in the shallow water also gives the maximum change of trim, and that above this speed the resistance soon becomes less than that experienced in deep water, and remains less at all higher speeds.

This effect is also illustrated by the trials of a "River" class destroyer of Thornycroft build, which attained 2·5 knots more in deep water than on the Maplin mile of 7 to 9 fathoms depth. These vessels were designed for a speed of 25 knots in deep water, and this approaches very closely to that for the worst possible results on this course, particularly at or about high tide.

Similar results have been obtained on the trials of Danish torpedo-boats and other high-speed craft in shallow water, by

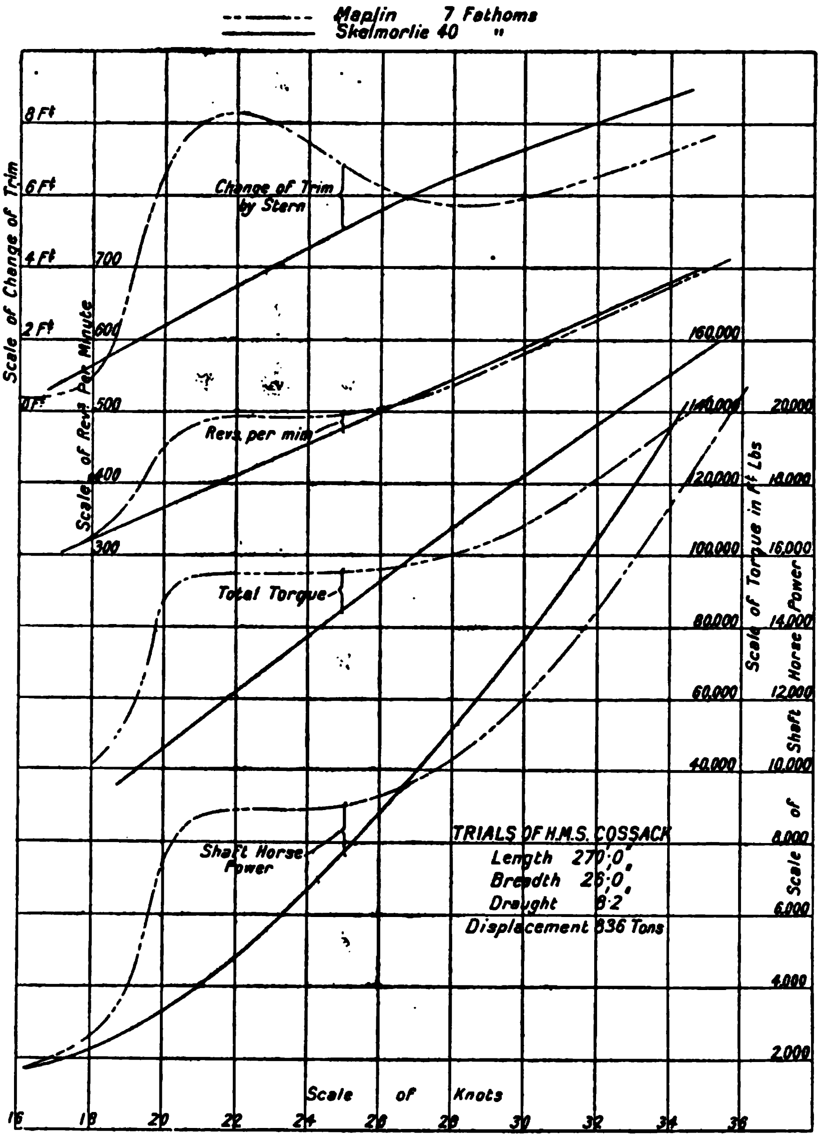


FIG. 28.—Effect of Depth of Water.
High-speed Vessel.

Herr Paulas in trials of the German torpedo-boat *S* 119, and by Mr. Yarrow in trials with H.M.S. *Usk*. All of these results on

full-sized ships show that these high-speed vessels, travelling in water of depth d in feet,

(1) experience their maximum resistance at a speed V, given by

$$V^2 = 11d,$$

as theory leads one to expect; the effect being more emphatic the shallower the water. The constant in the formula given is slightly lower at speeds which are high compared with the natural wave-making speed of the boat, and slightly higher at lower speeds.

(2) Above this critical speed the propulsive power required at first increases only very slowly compared with the increase in deep water, and ultimately the power required to drive the vessel in shallow water becomes less than that in deep water at the same speed.

Experiments with models of torpedo-boats published by Major Rota and Mr. Yarrow show very much the same phenomena, and agree with the formula for the critical speed derived from the full-sized vessels.

§ 62. Moderate and Low-Speed Vessels.—Shallow water phenomena are theoretically the same in their broad characteristics for all ships irrespective of whether their form is adapted to low or high speeds, and every vessel if forced to sufficiently high speeds (as can be done in model experiments) experiences the abnormal humps followed by depressions in its resistance. At very low speeds the increase of resistance is not very noticeable, but it becomes more and more emphatic for given depth of water as the speed increases, the growth of resistance becoming abnormal at a speed given by :—

$$V^2 = 5{\cdot}0 \times d.$$

At lower speeds the increment of resistance is largely that due to increased stream line velocity, the *abnormal* rise commencing as the depth of water becomes more favourable to wave-making.

Below this wave-making speed no general law can at present be given for the amount of the increase of resistance due to shallow water. The experiments of Professor Sadler and Mr. Taylor show

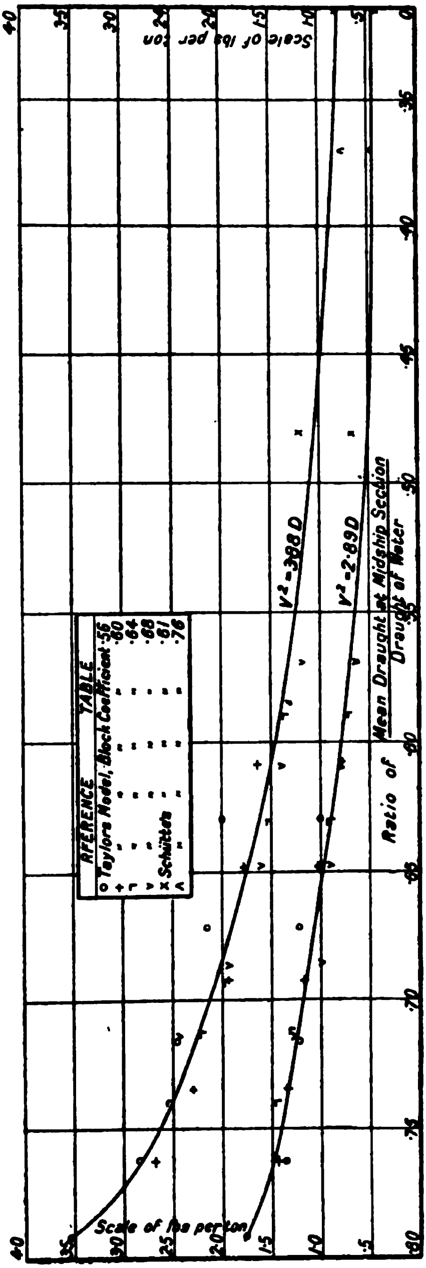


FIG. 29.—Additional Resistance in Shallow Water.

Moderate-speed Vessels.

that it is practically independent of the length of the vessel, and that it depends chiefly upon the ratio $\frac{\text{mean depth of midship section}}{\text{depth of water}}$. The greater the ratio the sooner this effect becomes marked, and the greater is the shallow water increment. Prismatic coefficient appears to have little influence at speeds below that given by the preceding formula. This can be seen from Fig. 29, which gives the results of some recent experiments with models in shallow water. The figure gives the *increase* in resistance per ton displacement of ship at two speeds, the ordinates being plotted to a base of $\frac{\text{mean depth of midship section}}{\text{depth of water}}$. Four block coefficients varying from ·56 to ·68 were tried with varying midship section areas. The particulars of these models are given in Table 17.

These experiments show that in shallow water there is no gain as regards resistance by adopting a large midship section area, when this means a deep draft, even though it gives better results in deep water.

Johann Schütte's experiments with models of an Atlantic liner and a cargo-boat support the above results, and it will be noticed that even at a speed given by $V^2 = 2{\cdot}88d$ there is a material increase of resistance for all depths of water tried in the experiments. For practical speeds Schütte's experiments show that the following depths of water are required in order to be entirely free from shallow water effects :—

Destroyer, more than 14 drafts.

Fast liner, about 7·5 drafts.

Cargo-boat, more than 5 drafts, but not more than 7.

The increase in power required to steam at speeds well below the critical speed for the depth of water is shown by the following trial results of two warships :—

H.M.S. *Edgar* in deep water steams at 21 knots, but in 12 fathoms with the same power she can do only 20·25 knots. The *Edgar* has a midship section area of approximately 1,150 square feet, mean depth of midship section of 22·5 feet, length between perpendiculars of 360 feet.

H.M.S. *Blenheim* in deep water steamed at 21·5 knots with the same power as developed only 20 knots in 9 fathoms. Her midship section area is 1,320 square feet, mean depth of midship section 25·5 feet, length between perpendiculars 375 feet.

One important effect of shallow water is the increased sensitiveness of the stream line flow. Mr. Taylor, in giving the results of his experiments, pointed out that unstable eddies caused the resistance to vary radically at high speeds, and even at low speeds the results required some fairing. This eddy formation must necessarily produce bad manœuvring in any ship working under the same conditions.

§ 63. Width and Depth of Channel.—Schütte has tested several forms in channels of varying width and depth, and in what may be called open and unlimited water. Taking first the case of a cargo-boat. The increase in resistance at any speed over the open water value was approximately the same for a broad and shallow channel as for a narrow and deep channel, provided the ratio $\frac{\text{depth of water}}{\text{draft of ship}}$ in the former and $\frac{\text{width of channel}}{\text{beam of ship}}$ in the latter were the same. This only applies to the ordinary low speeds at which such a vessel would run, when wave-making is not very important. Over this range of speed, the percentage increase of resistance was roughly constant for any given size of channel.

At higher speeds, experiments with models of an Atlantic liner and a torpedo-boat show—(1) the speeds at which humps occur in the curve of $\frac{\text{resistance}}{(\text{velocity})^2}$ are slightly lower than in open and deep water, but (2) these humps and the succeeding hollows become more emphatic the narrower the channel. Table 23, p. 135, gives the percentage increase in resistance found with the above two models at various speeds. The figures, however, are not very accurate, being obtained from small scale diagrams, but they serve to show that a width of channel of at least eight beams is required for the liner and ten beams for the torpedo-boat to get the same results as in deep water.

TABLE 23.

Effect of Width of Channel.

—	Number of Beams in Width of Channel.	Percentage Increase on Resistance at (P) Value.							
		2·0	1·75	1·5	1·25	1·0	·9	·8	·7
Torpedo-boat: Block coefficient ·45. Prismatic coefficient, about ·6	3·4	7·0	16·3	7·5	32·0	9·5	9·5		
	5·67	1·6	8·0	5·1	12·0	7·4	7·0		
	7·9	·4	1·6	—	4·7	2·5	1·6		
Atlantic liner: Block coefficient, ·61 Prismatic coefficient, about ·64	3	—	—	—	—	20·6	26·0	16·5	9·2
	5	—	—	—	—	1·6	6·7	4·3	4·1
	7	—	—	—	—	—	3·0	2·0	

NOTE.—The bad wave-making speed of the torpedo-boat in deep water is given by (P)=1·5. The highest service speed of the liner would be approximately (P)=·8.

When the channel is both narrow and shallow there would be the same tendency to form the shallow water wave, but the actual wave formed would be affected by the width of the channel and the speed at which the vessel was running—*i.e.*, whether this speed was one at which the ship tended to form waves in deep and open water. But the shallowness appears to be more important than the width, and would be the main factor in determining the extra resistance.

Some tests were made at Uebigau with a number of similar models of different sizes. These were tried in two channels, one of which had a cross-section area nine times greater than the other, both having a breadth twice the depth. These experiments gave the following results :—

(*a*) An area of channel 200 times the midship section area of the model had no effect up to speeds given by $V^2=5{\cdot}0d$. Above this speed the usual shallow water hump occurred at about $V^2=10d$, and at higher speeds the resistance was slightly lower in the channel than in open water. The increase of resistance at the hump was 4 per cent. for a model of draft equal to $\cdot05d$, and 12 per cent. for a model of draft equal to $\cdot1d$.

(*b*) An area of channel 51 times the midship section area of the model increased the resistance at all practical speeds, this increase being of the order of 8 per cent. at $(P) = \cdot 78$. With a fine form (prismatic coefficient ·6) the increase of resistance was continuous up to the critical speed for the depth, and exceeded 22 per cent. at this speed. With a fuller and deeper form (prismatic coefficient ·8) the resistance hump and hollow at $(P) = \cdot 8$ to $1 \cdot 1$ was exaggerated in the small channel, and the percentage increase of resistance over the open water value varied considerably in consequence. When the speed exceeded that given by $V^2 = 5d$ the resistance increased enormously. This model had a draft one-fifth the depth of water in the small channel, the draft of the former one being one-tenth.

(*c*) In a channel of cross-section area 22·8 times that of the model the resistance of the fine and shallow form was increased from 4 per cent. at quite low speeds to 45 per cent. at a speed given by $V^2 = 5 \cdot 0d$, and much higher figures above this speed. For the fuller and deeper form (draft$= \cdot 33 \times d$) the resistances were increased 12·5 per cent. and 30 per cent. at speeds given by $\frac{V}{d} = 1 \cdot 15$ and $2 \cdot 6$ respectively.

It will be seen that in a confined channel, the deep vessel suffers more than the shallow one, and that a cross-section area of channel about 200 times that of the vessel is required at all normal speeds, if the resistance is not to be affected by the boundaries. At low speeds a smaller size can be allowed, the necessary ratio of areas being about 150. In the previous pages it has been shown that there must be a depth of water equal to 14·0 to 7·0 drafts when there are no side boundaries, and a breadth of channel equal to 10·0 to 8·0 beams when the water depth is very great. When both side and bottom boundaries are present these numbers naturally increase somewhat, as shown by the above results.

§ 64. Barges and Canals.—It was demonstrated many years ago by Scott Russel that a barge could be towed in a narrow and

shallow canal much easier at high speeds than at low, and that at a certain critical speed which had to be passed to attain this result the resistance was remarkably high and the wash on the canal banks very destructive. This critical speed he found to be that at which a wave of translation proper to the canal depth would

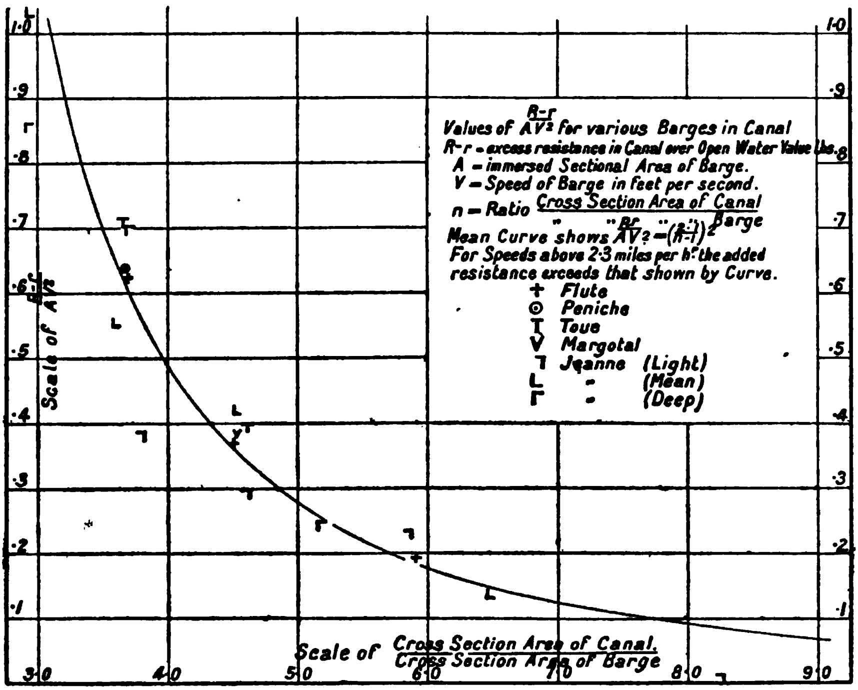


FIG. 30.—Additional Resistance in Canals.

Barge Forms.

travel, and is given by the formula in § 60. This speed is, however, too high for the usual run of canal-boats, being 8·1 knots for a depth of 6 feet.

In considering this matter it must be remembered that the usual type of canal-boat has a very high block coefficient (generally above ·9), which necessitates blunt ends. Irrespective of any effect of the restricted area of the waterway, however, these vessels are not suited for running at high speeds, owing to the growth of wave-making and eddy resistance. Moreover, in many

inland canals there is a definite speed limit which must not be exceeded. This speed has been fixed mainly with the idea of preventing the banks from being destroyed by the wash of the waves created by passing vessels, and averages about three* miles per hour, and it is at this and lower speeds that the effect of the restriction of area is of greatest interest.

Reliable data on the subject is scarce, but from such as is available the following conclusions may be drawn :—

(1) *Canals which are broad and shallow give worse results than deeper and narrower canals of the same sectional area.* This is due to the fact that a vessel requires a certain depth of water between it and the canal bed. If this depth of water is not available, the stream flow under the bottom becomes very unsteady and the resistance is increased due to the consequent increase of eddy-making and erratic motion of the vessel. This depth appears to be not less than 1·5 feet, and if possible 2·0 feet for canals of about 7·0 feet total depth.

(2) *The resistance in the canal depends very largely upon the ratio* $\frac{\text{area of canal waterway}}{\text{immersed section of boat}}$. The resistance in a canal may be divided into resistance in open water and added resistance in the canal. Experiments made by M. de Mas show that this added resistance for speeds up to 2·3 miles per hour may be written in the form

$$R-r=\left(\frac{2\cdot 1}{n-1}\right)^2 AV^2.$$

R and r being resistance in lbs. in shallow and deep water respectively;

A the immersed sectional area of the boat in square feet;

V the velocity in feet per second;

n the ratio $\frac{\text{(area of waterway).}}{A}$

* This figure does not apply to intersea canals, the speed limits for these being as follows:—Suez, 5·3 knots; Kiel, 8 knots light draft, 6·5 knots if draft exceeds 16 feet.

The spots in Fig. 30 are obtained from the experiments with the vessels whose dimensions are given in the following table, and the curve put through them represents the above equation :—

Table 24.

Dimensions of Barges and Values of $\frac{r}{AV^2}$ in Open Water.

Barge.	Length (feet).	Draft (feet).	Cross-section Area (square feet).	Dis-place-ment (tons).	Block Coeffi-cient.	$\frac{r}{AV^2}$ at Speeds (foot seconds)			
						1·64	3·28	4·92	6·56
Prussian boat	112	4·27	69	200	·93	·26	·24	·244	·26
Toue-boat	118·4	3·28	54	—	—	·59	·49	·47	·47
		4·27	70	226	—	·51	·41	·41	·42
		5·25	86	—	·97	·51	·38	·37	·39
Alma	124·6	5·25	86	286	—	·51	·39	·375	·40
Rene	99·4	5·25	86	226	—	·51	·38	·375	·40
Adrien	67·4	5·25	86	148	—	·51	·38	·375	·40
Flute	123	4·82	79	256	—	·78	·51	·42	·49
Peniche	125	5·94	97	337	·99	1·0	·71	·72	·77
Jeanne	99	3·28	53	—	—	·20	·43		
		4·27	70	—	—	·44	·39		
		5·25	84	—	—	·39	·35		
No. 2, Fig. 31	100	4·67	106	247	·82	—	·21	·18	·23
No. 8, Fig. 31		1·00	22	45	—	—	·49	·43	·36
No. 4, Fig. 31	100	4·05	92	247	·94	—	·39	·33	·42
		·87	20	45	—	—	·62	·57	·50

Notes.

Peniche is slightly rounded at each end, and the bottom cut away slightly at stem and stern.

The *toue*-boat has a square stern, but is cut away more than the *Peniche* at the bow.

In connection with the above, the following results obtained by Herr Gebers with box-shaped models are of interest. The boxes had a perfectly square section throughout, the breadth being equal to the draft. The values of $\frac{r}{AV^2}$ obtained were—

With length equal to ten beams, 1·16.
With length equal to five beams, 1·10.

This table also gives the values of $\frac{r}{AV^2}$ for the barges in open water, so that the percentage increase due to the canal may be

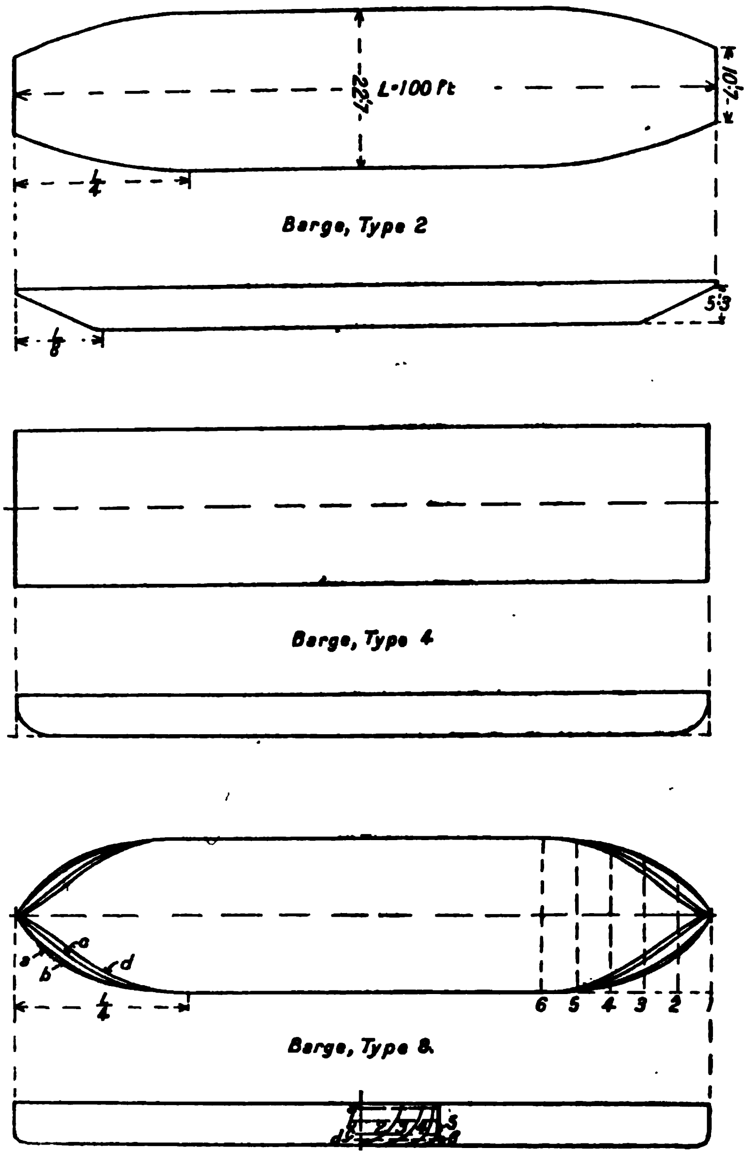


FIG. 31.—Barge Forms.

obtained. Incidentally the table shows the great decrease in resistance resulting from a slight fining of the bow of the barge.

The extra resistance which a barge experiences in a canal is partly due to the heaping up of water across the canal in front of the barge, and if a finer entrance was used on these forms the canal resistance would drop considerably, as the water could more easily pass to the rear. This advantage of a fine entrance would be felt in open as well as in restricted waters.

(3) *Any advantage due to its form which a barge may possess in open water, is to a certain extent masked in the canal, but is never obliterated.* In every case the canal boat which towed best in the river Seine was found to tow best in the canals, the difference in resistance of two boats at the same speed being roughly the same under both conditions. Taylor's experiments, already discussed, seem to indicate that this may not be true for much finer vessels, but although there may be limits to the above generalisation, they are so far removed as not to affect its application to the type of vessel considered.

(4) *The choice of form for a towed barge will depend largely on the character of the traffic, i.e.*, whether the tow is short or long, and whether there is a train of barges or only one. Taking the case of the short tow, Sadler's experiments show that a form as Type 2, Fig. 31, gives a good performance, either singly or in groups. But Type 4 carries a bigger load on given dimensions, and in groups, towed stem to stern, is little worse than Type 2.

For a long tow, Forms 2 and 8 are both reasonably good for resistance in deep and shallow water (see Table 24). A better form than either of the above for working singly, especially for deep water, can be obtained by using a little more rake of ends than that of Type 2, and by rounding the ends into the sides and bottom. For long trains of barges, towed stem to stern, Form 4 is as good as any, and for given displacement is decidedly the best in very shallow water. But taken singly, even in shallow water, either Types 8, 2, or 2 modified, are better than Type 4.

(5) Where resistance is of any importance, steel barges are much more economical than wood, as the wood surface quickly roughens and splinters, causing a large increase in resistance,

this increase, with a very moderate roughening, being some 20 per cent.

(6) In deep water, the power required to tow a train of barges increases, roughly, with the number of barges. In shallow water a long train is much better than a broad one. With only a little water between barge bottom and river bed, a long train, towed stem to stern, will have little more resistance than a short one.

PART II

THE SCREW PROPELLER

CHAPTER XVI

SCREW PROPELLER NOMENCLATURE AND GEOMETRY

§ 65.—The **driving face or front** of a screw blade is the surface which is seen by an observer looking from aft to forward.

The **back of a blade** is that which is seen by an observer looking from forward to aft.

The **leading edge** of a blade is that which leads or cleaves the water when the vessel is going ahead under the action of its own propellers.

The **trailing edge** is the other edge of the blade.

The **developed area** of a blade is the actual area of the blade surface irrespective of its shape.

The **developed area of the propeller** is the sum of the developed areas of its blades.

The **projected area** of a blade is the area enclosed by perpendiculars from the edge of the blade on an athwartship plane.

The **projected area of the propeller** is the sum of the projected areas of its blades.

The **disc area** of a propeller is the area of the circular section swept by the blade tips, and equals $\frac{\pi}{4}$ (diameter)^2.

The **disc area ratio** is the fraction $\dfrac{\text{developed area of screw}}{\text{disc area}}$.

The **blade width ratio** (b) is the fraction

$$\frac{\text{maximum width of blade along its surface}}{\text{radius of propeller}}.$$

The developed surface of many blades takes the form of an ellipse having the radius of the propeller as the major axis, the ellipse necessarily being cut away at the boss. In this case b is the ratio of minor to major axis.

The **mean width ratio** is the ratio of the mean width of the blade clear of the boss, to the diameter of the propeller.

The **root thickness** of a blade is its maximum thickness at the boss square to its face, ignoring any rounding at the joint of blade and boss.

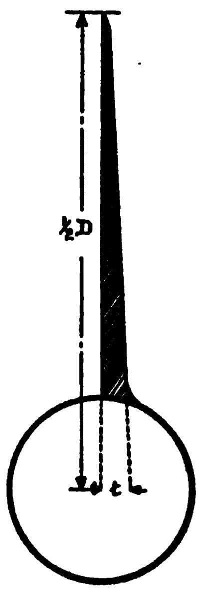


FIG. 32.

The **blade thickness ratio.** If the median lines on the face and back of a blade be produced to the centre of the propeller where their width apart is t (see Fig. 32), then $\frac{t}{D}$ is called the blade thickness ratio.

Rake. A propeller blade is said to be raked forward or aft, according as the centre line of the blade at the tip is forward or aft of the centre line at the root. The rake is usually expressed in degrees inclination to the transverse plane, of the line joining these two points.

A **helicoidal surface** is that traced out by a line AB (Fig. 33), of which one point A moves at uniform rate along a line CD whilst AB is revolving around it at uniform rate.

A **blade of uniform pitch** is one whose face is a portion of a true helicoidal surface, and its *pitch* is the distance A advances whilst AB makes a complete revolution.

A surface is said to have *increasing pitch* when the pitch at the trailing edge is greater than that at the leading edge, *i.e.*, when the generating line AB (Fig. 33) advances along the line CD from leading edge to trailing edge with increasing speed whilst it continues to revolve at uniform speed.

If the point B advances in the direction CD faster than the point E, then the pitch at B is greater than at E, or the pitch is said to *increase towards the tip*.

The **face pitch of a propeller** is the mean of the face pitches of all its blades.

The **effective** or **analysis pitch of a propeller** is that calculated from the revolutions of no thrust. Thus, if at speed v in feet per minute the propeller develops no thrust when the revolutions are r per minute, the effective pitch is given by $\frac{v}{r}$. When the

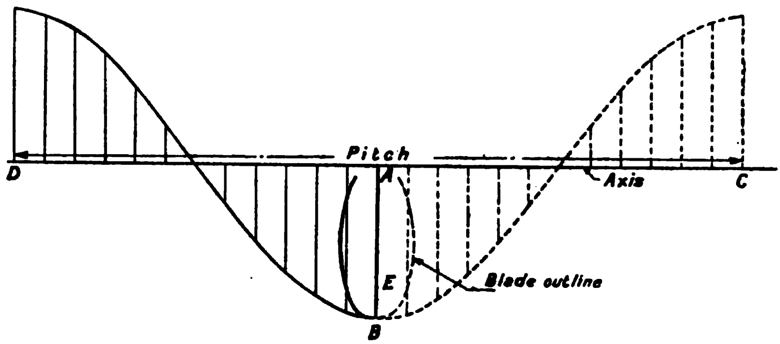


FIG. 33.

thrust of the propeller is zero, the slip calculated from this pitch is also zero.

Let P =pitch of the propeller face in feet;
D =diameter of propeller in feet;
N =number of revolutions in hundreds per minute;
V =speed of the ship in knots;
V_1=speed of advance of screw through wake water in knots;
N_o=revolutions for no thrust, in hundreds per minute;
P_r =effective pitch of the propeller.

Then apparent slip

$$=s_a=\frac{PN-V\times 1{\cdot}013}{PN}$$

effective slip

$$=s_r=\frac{N-N_o}{N}$$

and

$$N_oP_r=V_1\times 1{\cdot}013.$$

Unfortunately the value of P_r is very difficult to determine without careful trials, and in consequence it is not generally used.

The apparent true slip or as it is usually called the slip is given by

$$s=\frac{PN-V_1\times 1{\cdot}013}{PN}.$$

This is the real slip of the propeller if the actual or effective pitch were the same as that of the propeller face.

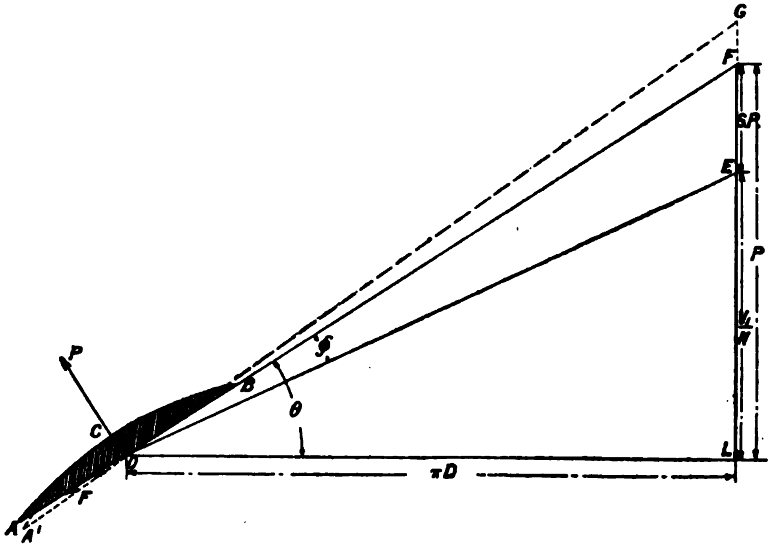


FIG. 34.

For definitions of wake, thrust deduction, hull efficiency, etc., see § 71.

Slip angle and pitch angle. Consider any portion of the blade surface of a propeller of pitch P and let the diameter of this portion be D. In Fig. 34 draw

$$OL=\pi D \text{ and } FL=P$$

Then if the triangle OFL were cut out and wrapped around a cylinder of diameter D keeping the line FL parallel to the axis of the cylinder, the line OF would form a helix, and a section of the blade would be as shown by $AOBC$.

The angle θ is called the pitch angle of the section. If the section is working at slip s, make

$$FE=s(FL)$$

where

$$s=\frac{P-\frac{V_1}{N}}{P}$$

so that

$$EL=\frac{V_1}{N}.$$

The angle EOF is called the slip angle of the section, and is the inclination of the plane of the section to the direction of its motion.

Since FE and EL are the same for all parts of the same screw provided it has uniform pitch, it follows that ϕ is constant for all parts of the blade having the same diameter, but decreases as D increases. If the pitch of the trailing edge is greater than that of the leading edge, draw LG equal to this new pitch. Then the slip angle of the trailing edge is GOE, and the blade section at this diameter is as shown by A_1OBC.

CHAPTER XVII

THEORIES OF THE SCREW PROPELLER

§ 66.—A screw propeller is a means of pushing a ship forward by thrusting water aft. For this two things are required :—

(1) An unobstructed and free supply of water ;

(2) A mechanism which shall only thrust aft, and not rotate the water, or waste energy by shock.

These conditions are ideal, but the more nearly they are approached the more efficient becomes the propeller. The method by which the propeller thrusts the water aft is known only in a general way. Probably the nearest approach to the truth is the ideal propeller theory advanced by Mr. R. E. Froude.

In this theory it is assumed :—

(*a*) The propeller mechanism is frictionless and that no energy is lost in heating the water or in causing eddies.

(*b*) The race column is complete and has a uniform velocity.

(*c*) The pressure in the outer layer of the race is the same as that of the surrounding water, *i.e.*, the same as it had before the propeller acted upon it.

(*d*) There is an unlimited supply of water.

(*e*) That at the front the streams are drawn or sucked into the propeller, and at the rear are thrust backwards owing to an excess pressure, the streams contracting towards the race owing to the increase in velocity of the particles.

Writing V_1 = the relative velocity of the screw to open water ;
U = the sternward velocity imparted to the water ;
T = the thrust of the propeller ;
v_D = velocity of the water at the propeller ;
Q = the quantity of water acted upon per second,

we have

$$T=\frac{w}{g}Q\times U \quad . \quad . \quad . \quad . \quad (1)$$

and the work expended per second in slip

$$=T(v_D-V_1)=\frac{1}{2}\frac{w}{g}Q\times U^2;$$

therefore

$$(v_D-V_1)=\frac{U}{2} \quad . \quad . \quad . \quad . \quad (2)$$

that is, one half of the sternward velocity has been received by the water before it reaches the propeller.

This velocity is largely due to the suction of the propeller.

Taylor has explored the pressures existing around a working screw and has found that there is a considerable reduction of pressure in front of it—appreciable to 1·5 diameters ahead of it. This reduction, expressed in terms of the screw thrust, was broadly speaking independent of the pitch ratio and slip of the propeller.

The useful work done by the propeller $=T\times V_1$, the total work done by the propeller $=T\times v_D$, and the efficiency

$$\eta=\left(\frac{V_1}{V_1+\frac{U}{2}}\right) \quad . \quad . \quad . \quad (3)$$

This may be turned into a more useful form by writing

$$\text{slip } s_t=\frac{U}{V_1+U},$$

and equation 3 becomes

$$\eta=\left(\frac{1-s_t}{1-\frac{s_t}{2}}\right) \quad . \quad . \quad . \quad . \quad (4)$$

If the race is rotational, assuming that the angular and translational velocities imparted are the same throughout the race column, the equations for thrust and efficiency become

$$T=\frac{Qw}{g}\left(U-\frac{r^3\omega^2}{4(V_1+U)}\right) \quad . \quad . \quad . \quad (5)$$

and

$$\eta=\frac{1-s_t}{1-\frac{s_t}{2}}\left(1-\frac{s_t r^2\omega^2}{4U^2}\right) \quad . \quad . \quad . \quad (6)$$

where r is the external radius of the race column and w the angular velocity of the race.

It will be seen that the loss due to rotation of the race consists in a term $\frac{s_t}{4}\left(\frac{r\omega}{U}\right)^2$. The bracketed part of this is the tangent of the inclination of the spiral path of the race particles with the line of motion. Fig. 35 represents the motion taking place.

FIG. 35.

These formulæ 4 and 6 give the ideal efficiencies which are possible under the assumptions made, and show in a general way the disadvantage of rotation of the race, which becomes more noticeable when for any reason the slip becomes large. This may explain the advantage which is sometimes derived in a screw propeller by fitting guide blades at the rear of the screw.

If D=the diameter of the propeller, from equation 2,

$$Q=\frac{\pi D^2}{4}\left\{V_1+\frac{U}{2}\right\}$$

and multiplying equation 1 throughout by V_1

$$TV_1=\frac{\pi}{4}\frac{w}{g}D^2V_1^3\frac{s_t\left(1-\frac{s_t}{2}\right)}{(1-s_t)^2} \text{ foot-pounds,}$$

which is a formula for the maximum power a propeller will deliver.

These formulæ do not represent the actual case of a working

propeller owing to the fact that the assumptions made are only partially realised in practice. Every fluid is frictional, eddies are formed, the ship's form impedes the flow of the water, the completeness of the race column depends upon the form and disc area of the propeller, and for all these reasons the actual case differs from the ideal.

Provided that the blades of a screw did not interfere with each other's action (which they do), and that their thrust was proportional to their slip angle, the race velocity U would be proportional to the slip of the blades, and in this case the term s_t would be a measure of the slip of the screw. The formulæ may therefore be regarded as giving ideal thrust and efficiency in terms of slip (effective), provided the above assumption as regards the actual propeller thrust is true—which is the case for all moderate slips.

§ 67.—Of the other theories of the screw propeller three only need be mentioned—viz., those of Rankine, Greenhill, and W. Froude.

Rankine assumed that the propeller in thrusting the water aft gave it a spiral motion and that there was no change of pressure in the race—*i.e.*, the streams contract suddenly in passing through the screw—and that the race forms a complete column. The centrifugal action causes a change of pressure and the loss of thrust due to this can be calculated, as also can the loss by skin friction.

Greenhill assumed that the propeller gave no sternward but only rotary velocity to the water—*i.e.*, there was no contraction of the streams—and that the thrust was due to the increase of pressure in the race. This rotary motion, however, produces a pressure at the axis differing from that at the tip of the screw, except for one particular slip (67 per cent.), and actually, for equilibrium, the outer streams must be accelerated and the inner retarded at all moderate slips.

Neither of these can be put to any practical use. They neglect the effect of form and dimensions of blade, pressure limits, and limitations of water supply, and to obtain the effect of these items one is driven to experiments.

W. Froude advanced the theory that an element of a screw blade may be considered as a small inclined plate moving through water, and its efficiency and thrust can be calculated by the use of the known normal and frictional forces on such plates.

Referring to Fig. 34, the velocity of any element of a propeller blade through the water is given by

$$\frac{V_1}{\sin(\theta-\phi)}.$$

The normal force on it

$$=P=a(\delta A)\frac{V_1^2}{\sin^2(\theta-\phi)}\sin\phi;$$

the tangential force

$$=F=f(\delta A)\frac{V_1^2}{\sin^2(\theta-\phi)}.$$

The thrust of the element along the axis

$$=T=P\cos\theta-F\sin\theta.$$

The transverse force

$$=M=P\sin\theta+F\cos\theta.$$

Hence

$$T=a(\delta A)V_1^2\frac{\sin\phi\cos\theta-\dfrac{f}{a}\sin\theta}{\sin^2(\theta-\phi)}.$$

And the efficiency

$$\eta=\frac{TV_1}{M\times\begin{pmatrix}\text{rotary}\\ \text{velocity}\end{pmatrix}}=\frac{\tan(\theta-\phi)}{\tan(\theta+\phi_1)},$$

where

$$\tan\phi_1=\frac{f}{a\sin\phi},$$

and a and f are the coefficients of normal and frictional force on a plane surface.

Curves of efficiency obtained in this way agree in general character with those obtained by experiments with model screws. The method, if extended to a whole blade, becomes very unwieldy and throws no additional light on the general theory of the propeller. It is tacitly assumed that there are no limitations to the

water supply, that interference of one blade with another is *nil;* also the fact that additional area becomes useless when a complete column is formed behind the screw is ignored. A simple and useful variation of this theory has been suggested by A. Mallock. Referring to Fig. 36,

If OX is the path along which the plane is moving (O to X), de=the resistance of the plane measured along $OX=R$, ea=the reaction normal to $OX=L$.

If this blade is a part of a screw whose longitudinal axis is OY, the reaction of de and ea along the axis=ba, and reaction normal to the axis=db.

As the plate moves along OX, forces normal to this line do no work, and the efficiency is the ratio of the resolved parts of the forces ba and db along OX

$$\eta=\frac{fe}{fd},$$

or

$$\eta=\frac{bc}{bd}=\frac{\tan a}{\tan (a+\beta)}.$$

Since

$$\tan \beta=\frac{R}{L}=\frac{\text{resistance}}{\text{lift}},$$

the efficiency for any pitch ratio and slip can be calculated for any type of blade whose $\frac{\text{resistance}}{\text{lift}}$ $\left(i.e., \frac{R}{L}\right)$ ratio is known. There is a large amount of data in existence giving the value of this factor for various types of blades, and this formula will be employed in later sections to examine the effect of shape, etc. It has the advantage that interference of one blade with another, and effect of shape, can be taken into account by the proper choice of $\frac{R}{L}$ values.

The absence of any theory which will take complete account of working conditions, forces the designer to depend upon his general experience or to have recourse to experiments. Reliable information as to the action of a screw in a given ship is difficult to

obtain, unless those making the steam trials go out of their way to get it. Times of delivery and financial considerations limit the possibility of trying several screws on the same ship, and unless this becomes necessary owing to deficiency of speed, such tests are seldom made and still more seldom published. The designer's knowledge is therefore more or less limited, and here

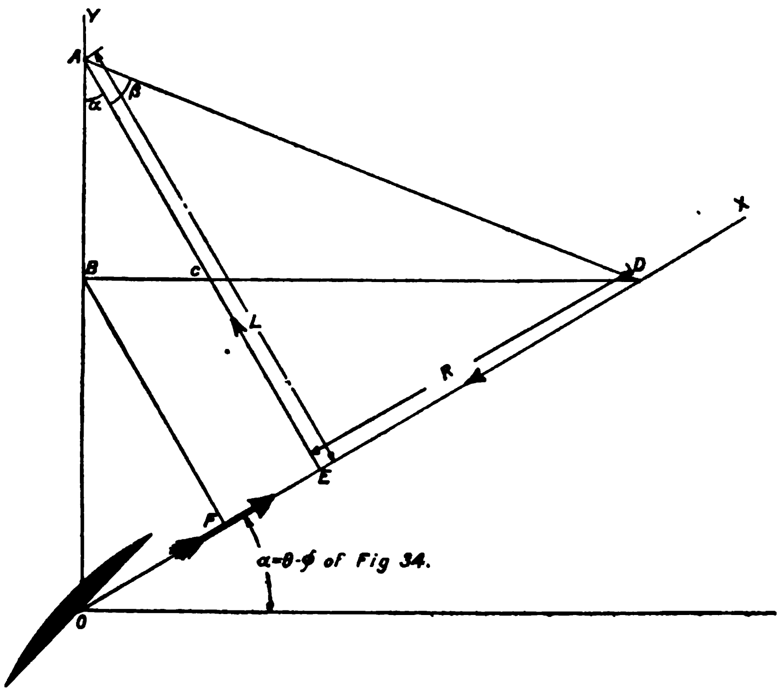


FIG. 36.

more than anywhere experiments on a small scale should be advantageous.

§ 68. Applicability of Small Scale Experiments to Full-sized Screws.—But for experiments on a small scale to be of use two things are required :—

(*a*) A law enabling results for a full-sized propeller to be predicted from the model.

(*b*) Exactly similar conditions in the model as in the ship, or a means of allowing for the difference.

First consider the case of two screws advancing into still water, the one of diameter D at velocity V and revolutions per minute N and the other of diameter d at velocity v and revolutions per minute n, where

$$\frac{D}{d}=l \text{ and } v=\sqrt{l}\times v.$$

Then if both screws are similar and *work at the same slip* the flow past each screw will be similar, the difference being one of scale only, and this similarity holds for any similarly related speed provided the race columns are equally complete or incomplete. For the slips to be the same,

$$n=N\sqrt{l}.$$

From the proof of the law of mechanical similitude given in § 8, it will be seen that under these circumstances the two screws will be giving thrusts T and t, which are related by the equation

$$T=l^3\times t.$$

Provided the losses in friction and eddy-making vary as the $(\text{speed})^2$, this relation still holds between the large and small propellers. If the propellers are near the surface the surface deformation will be "similar," since the pressures vary as the scale, and the above relations still hold good.

If the screw sets up a rotation in the race, for similar hydrodynamic conditions the angular velocity of the race should vary as

$$\frac{\text{velocity}}{\text{diameter}}, \text{ i.e., as } \frac{1}{\sqrt{l}},$$

or as the revolutions at the same slip ratio. Under these conditions the formula for thrust given above still holds. This argument is quite sound for the ideal case of the complete race column, but with the incomplete column the race rotation and translational velocity is not the same for all particles in the column, and although this is the case in both large and small screws, and with perfect streaming flow the action would be the same in both cases, there is no guarantee of this perfect flow. The extent to

which this possible loss of similarity may affect the above law of comparison can best be judged by experimental results.

§ 69. Experimental Tests of the Law of Comparison.—Mr. Taylor has tried in the Washington tank screws varying in diameter from 8 inches to 24 inches. He found some slight variation in the value of $\frac{T}{D^2V^2}$ and efficiency at the same slip and corresponding velocities, as the size was varied. These tests were not made under quite similar conditions, as the centres of the screws were all at the *same* immersion, 12 inches—*i.e.*, the immersion of the blade tips became smaller with the larger propellers. Preliminary experiments with the 16-inch screw having the blade tips at varying immersion, showed a very slight reduction of $\frac{T}{D^2V^2}$ with reduced immersion. The variation found with the different sized screws at *fixed* immersion was of the same character and order as the above effect due to immersion, and the latter possibly accounts for the former in these experiments. The variation was only 2 per cent., and the law of comparison holds more closely than the results indicate, and may be accepted for the range covered by the experiments. Other experiments made by Mr. Taylor with the 16-inch diameter screw at various speeds showed that the thrust at given slip varied as $(\text{speed})^2$ over a considerable range of speed, as theoretical considerations lead one to expect.

Herr Gebers' experiments with models of a turbine type of screw varying in diameter from 12 to 3 inches gave practically the same efficiency at corresponding speeds and slips, and the thrusts calculated for the largest screw from the measured thrusts with the smaller ones came within 3 per cent. of that obtained by experiment, over the useful range of slip.

An important test of the law has recently been made by experiments in air with exactly similar propellers of 15 feet and 2 feet diameter, the larger one on the whirling arm at Messrs. Vickers, Maxim and the smaller one on the whirling arm at the National Physical Laboratory. The tests were made at corresponding

speeds, and the thrust and horse-power for the smaller propeller were estimated from the tests of the larger propeller, and compared as follows :—

Item.	Thrust in lbs.	Horse-power Absorbed.	Efficiency.
Deduced from the full-size propeller by the law of comparison . . .	2·05	·1128	·64
From experiments with small model	1·97	·1115	·62

The falling off in the small model is slight and may be neglected. For this case the law undoubtedly held good.

M. Dorand has compared similar propellers of different diameter in air, under very dissimilar conditions to the above. His propellers were 6·2 and 14 feet diameter respectively and practically the same in design. The results show a difference in comparative thrust of only 2 per cent., and the efficiencies were almost identical. Although these tests were made in air, they lend considerable weight to the practice of using the law for propellers in open water, *i.e.*, with no ship in front of them.

§ 70.—The second condition which must hold if the results of model propeller tests are to be of use in predicting results for full-sized screws is—all the conditions in the model tests should be similar to those holding in the ship, or there should be some known method of allowing for the difference. Screw experiments may be divided into two quite distinct classes, viz., those made to *test the screw* and those made to test the reaction between the screw and model. The former are made in open or undisturbed water, the latter with the screw of the correct size in its correct position at the stern of the ship. In the latter experiments the object is to measure two primary things—

(*a*) The amount by which a screw situated in a given position augments the resistance of the model when it is itself producing a thrust sufficient to propel the model.

(*b*) The average speed of the streams in which the screw is working, *i.e.*, to obtain the wake fraction.

With regard to both *a* and *b* one condition is essential, *i.e.*, that the position and diameter of the model screw shall be to scale for ship.

This is all that is really necessary from the hydrodynamic point of view, and the pressure gradients and accelerations produced at the stern of the model will then be similar to those in the ship. In other words, both ship and model will experience the same increase in resistance due to the action of the screw. But with the wake the same cannot be said. It can confidently be stated that the wake of a model so far as this is due to skin friction is greater than that for the ship. In the majority of cases some portion of the propeller works in the frictional belt, and a portion of the area of the model screw will be working in this frictional wake, and the total or mean wake obtained for the whole area of the screw will be exaggerated by this to some extent, so far as the result is applicable to the ship. If the screws are well removed from the hull, the wake is largely due to stream line or wave action, and in this case the difference between model and ship conditions is negligible. But the smaller the screws and their clearance from the hull become, the more the difference must be felt, and some correction of the result should be made to obtain the true wake for the ship with similar screws. In single-screw ships this must be the case, as the propeller then works in a large area of the frictional belt.

As in the general run of ships the error is probably not a great one, and in many cases must be small, it is the wiser course to accept the results than to hazard a correction which may lead farther from the truth.

Leaving the question of shaft web and bracket influence on one side, it can therefore be said that as the necessary conditions of similarity hold fairly closely between model and ship, the augment of resistance due to the screw obtained with the model holds for the ship, and generally speaking the same may be said of the wake.

CHAPTER XVIII

THE ELEMENTS OF PROPULSION

§ 71.—The whole question of the propulsion of the ship requires the consideration of engines, propeller, and hull, the net propulsive efficiency being the product of the efficiencies of these three things. The hull has been considered in the earlier chapters. The engine efficiency depends largely upon the type of engine used, whether it is a triple or quadruple expansion reciprocating engine, a turbine, gas, oil or electric engine, or any combination of these. The type of engine chosen affects the revolutions of the engine shafts, and in this respect also affects the screw, but the latter is otherwise independent of the engine and may be considered as a distinctly separate subject.

The propulsive force exerted by the propellers of a ship when the latter is travelling at a uniform speed, must be exactly equal to the resistance the ship experiences at that speed. The resistance of a ship when towed is due to the motion which it produces in the water as it passes through it. This water motion is to a considerable extent longitudinal, and shows itself abaft the stern in what is commonly called the "wake of the ship" or the current of water which follows it. If the ship is propelled by its own screws these cause considerable changes in the pressure of the water around the stern, whereby its resistance and therefore the screw thrust necessary for propulsion, become greater than the tow-rope resistance. Not only does the action of the screws affect the ship's resistance, but the presence of the ship affects the working of the screws. These have to work in this wake water, and their action will therefore depend to some extent upon the magnitude and direction of the currents in that portion of the wake on which they act.

The net efficiency of a screw as a propelling agent will depend upon two factors, of which one expresses the efficiency of the propeller itself if working in undisturbed water under similar conditions as regards thrust and revolutions. The other expresses the modification of this efficiency due to the action of screw upon ship, and ship upon screw. Mr. R. E. Froude, to whom this method of treatment is due, calls these two factors "the screw efficiency proper" and the "hull efficiency" respectively. It will be seen later that for many purposes these two factors may be regarded as separate elements, and this subdivision simplifies the work both of the designer and the experimenter.

Consider the case of a ship propelled by a screw.

Let T = the thrust of the screw in lbs.;
N = revolutions in hundreds per minute;
V = the speed of the ship in knots;
R = resistance of the ship in lbs. at the speed V with no screws, *i.e.*, the tow-rope resistance;
S = shaft horse-power delivered to the screw when propelling the ship;
E = the effective or tow-rope power, including bossing and keel resistance.

Suppose the screw to be now removed and set working in undisturbed or "open water," the revolutions being maintained the same but the speed of advance V_1 adjusted so that the screw develops the same thrust as it did behind the ship. Let S_1= shaft horse-power absorbed by it under these conditions. So far as the development of thrust is concerned, the screw behind the ship might equally well be advancing into undisturbed water of velocity V_1, and this velocity is usually taken as the mean velocity of the water past the screw in the wake of the ship. If the following current left by the ship were of uniform speed throughout the area operated upon by the screws, the above would be quite true, and the power required to drive the screw in open water at speed V_1, or behind the ship at speed V would be the same. It is a fact proved by many experiments on screws working behind

models and in open water, that the shaft horse-power of a screw propelling a model differs extremely little from that of the same screw in undisturbed water under the above conditions.

The **propulsive efficiency of the screw** is given by

$$\eta = \frac{\text{tow-rope power } E}{\text{shaft horse-power } S}$$

which may be split up into the factors given in the following equation :—

$$\eta = \frac{E}{S} = \frac{RV}{S} = \left(\frac{R}{T}\right) \times \left(\frac{V}{V_1}\right) \times \left(\frac{TV_1}{S_1}\right) \times \left(\frac{S_1}{S}\right).$$

The first bracketed term here is the ratio of the tow-rope resistance of the ship to the thrust of the screw.

If t is the fractional measure of the excess of the screw thrust over the tow-rope resistance, t is called the **thrust deduction fraction**, and is given by

$$1 - t = \frac{R}{T}$$

The second bracketed term is the ratio of the velocity of the ship to the mean velocity of its wake water taken over the screw disc, and writing

$$V = V_1(1 + w)$$

w is called the **wake fraction,** and denotes the fractional excess of the velocity of the ship over that of its wake water at the screw.

The third term is merely the efficiency of the screw in open water.

The fourth term is the relative measure of the power required to develop the screw thrust in open water and in the disturbed water behind the ship. This is known as the **relative rotative efficiency**.

The propulsive efficiency of any screw can therefore be written :

$$\eta = \begin{pmatrix}\text{screw efficiency} \\ \text{in open}\end{pmatrix} (1 - t)\,(1 + w) \begin{pmatrix}\text{relative rotative} \\ \text{efficiency}\end{pmatrix}.$$

The speed of advance of the screw through the water in which it is working is equivalent to a speed V_1 in undisturbed water, and

the work which it does in propelling the ship is measured by the product

$$T \times V_1$$

This is called the thrust horse-power of the ship.

The product $R \times V$ is a measure of the effective horse-power of the ship, and the net efficiency can therefore be written in a new form thus :—

$$\eta = \frac{\text{E.H.P.}}{\text{T.H.P.}} \times \begin{pmatrix}\text{screw efficiency} \\ \text{in open}\end{pmatrix} \begin{pmatrix}\text{relative rotative} \\ \text{efficiency}\end{pmatrix}.$$

This first term is defined as the **hull efficiency of the ship,** and since the latter term is generally unity the efficiency can be written

$$\eta = \text{hull efficiency} \times \text{screw efficiency in open},$$

where

$$\text{hull efficiency } h = (1-t)(1+w).$$

The net propulsive efficiency of the ship (e) is the product of the propulsive efficiency of the screw and the efficiency of the engine including thrust block and shaft friction, and can be written

$$e = \begin{pmatrix}\text{screw efficiency} \\ \text{in open}\end{pmatrix} \begin{pmatrix}\text{hull} \\ \text{efficiency}\end{pmatrix} \begin{pmatrix}\text{engine} \\ \text{efficiency}\end{pmatrix}.$$

The cost of propelling the ship any distance depends equally on each of these terms. Of these the most important and most liable to variation is the screw efficiency, and this is considered in the next section.

CHAPTER XIX

SCREW PROPELLERS IN OPEN WATER

§ 72.—Several broad conclusions can be drawn from the various theories of the action of propellers, and the considerable number of experiments which have been made on model propellers, and a few on large-sized propellers, enable these conclusions to be compared with empirical results. Of these the most important are the following :—

(*a*) The thrust and efficiency of any given screw, advancing through the water at any given speed and turning at various revolutions, will depend upon the slip ratio corresponding to the revolutions at any given moment.

(*b*) The thrust of a given screw working at a given slip varies as the square of the speed of advance through the water.

(*c*) The thrust at given speed of advance and slip ratio, will vary as the square of the diameter, for screws exactly similar in design.

(*d*) The efficiency at any slip ratio is unaffected by variations of speed or of size of screw.

Setting aside all question of cavitation, experiments show that the first three of these conclusions are true for actual screws, and although the efficiency varies slightly with speed, for all ordinary purposes this variation is small and can be neglected. It follows that the thrusts obtained from all screws of similar design at any given slip will be connected by the formula

$$\frac{T}{D^2V_1^2}=c,$$

where c is a constant depending on the slip and design of screw. Its variation with pitch, diameter, blade area, thickness of blade, and any other feature is one of the main and most difficult problems in propeller design. No theory yet formulated will give reliable quantitative values for pressure or efficiency. The only useful theory that will give relative efficiency values is that due to Mr. Mallock (see § 67). By the use of this theory a fair guide can be obtained as to what may be expected from any variation in design, and, failing experimental results on any point, it has been used in this way when dealing with the subject.

Experimental Results.—In using experimental results close attention must be given to the experimenter's method of tabulating and giving data, and more particularly as to how he defines the various terms used. The absence of uniformity of definition is responsible for no small amount of the doubt that has been thrown upon experimental results. Mr. Taylor uses the face pitch for both pitch and slip ratio. So also does Herr Gebers. Mr. Froude uses a pitch calculated from the revolutions and speed of advance so that slip shall be zero when thrust is zero. There can be no doubt that the latter is the more correct, as the back of a blade is just as important as the front, and should be taken into account in giving the blade its pitch value. Moreover, this method is more in accord with sound theory, and is less likely to give contradictory results. In applying results obtained by any model screw experiments to full-size screws, the pitch ratios obtained require multiplying by some coefficient, and Mr. Froude has stated that pitch ratios obtained by his method of analysis should be taken as 1·02 times the nominal or driving face pitch for the ship. This result, which is based upon experience in the analysis of ship trials, is better than any estimate based upon the relation of face to real pitch. This method has the advantage, too, of taking account of differences in boss arrangement and in blade thickness. For these reasons, and because the results are more compact, Froude's method has been adopted throughout the book, and the constants given in his paper read before the Institution of Naval Architects in 1908 have also been adopted.

Of the experimental results published, Froude's, Taylor's, and Durand's are the most important. To a large extent they cover the same ground, and when analysed in the same way corroborate each other as regards defining the effect of variations of pitch, width of blade, etc. The screws used by them differ mainly in blade thickness ratio and in the size of screw, and the effect of these factors will be considered in detail later.

§ 73. Froude's Experiments on Model Screw Propellers.—These experiments were made in open water, the screw being mounted on the fore end of a driving shaft so that there was no obstruction, either in front of it, or for some distance to the rear.

The diameter of screw was, uniformly, ·8 foot.
The boss diameter was, uniformly, ·91 inch.
The root thickness was, uniformly, ·27 inch.
The immersion to centre of screw was ·8 of the diameter.
The speed of advance was 300 feet per minute.

The face pitch ratios covered a range from ·8 to 1·4; the disc area ratio varied from ·3 to ·75.

Four-bladed and three-bladed propellers were tested. The developed blade shape was elliptical for all the four-bladed screws. The three-bladed included the elliptical and the wide-tip pattern. For the elliptical pattern the developed outlines were ellipses of major axis equal to propeller radius (see Fig. 42). The wide-tip blades were formed from these by making at any radius$=r$

$$\frac{\text{wide tip width}}{\text{elliptical width}}=\frac{1}{2}+\frac{r}{R},$$

where R equals the propeller radius.

Each screw was tested through a range of slip ratios, and thrust, revolutions, and turning moment were recorded. Special experiments were made to obtain the friction of the apparatus and the resistance of the frame and shaft carrying the screw.

The thrusts obtained were expressed in a formula

$$\frac{T}{D^2V_1^2}=B\left(\frac{p+21}{p}\right)\times\frac{1{\cdot}02\,s\,(1-{\cdot}08\,s)}{(1-s)^2}, \quad . \quad . \quad (7)$$

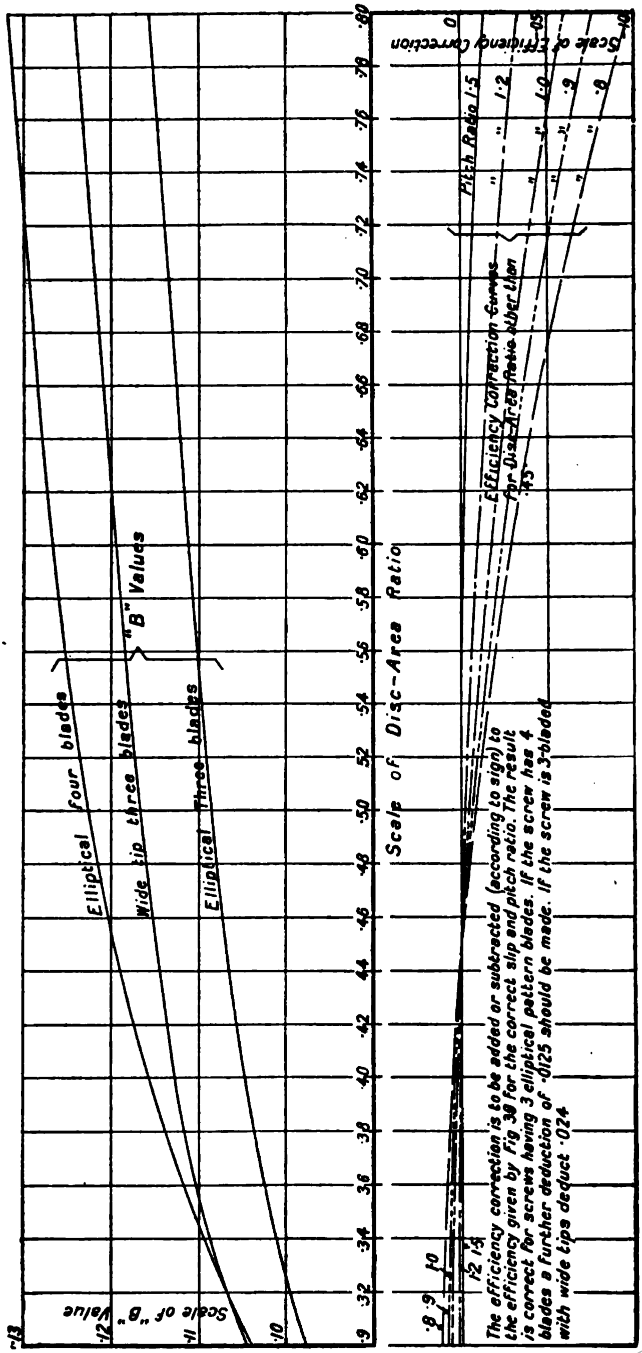


FIG. 37.—Variation of Efficiency, and Thrust Factor "B" with Disc Area Ratio.

where

T is the thrust in lbs.;

V_1 is the speed of advance in hundreds of feet per minute;

p is the effective or analysis pitch ratio;

B is a blade factor depending upon the number and type of blades and the disc area ratio.

Curves of B value obtained are given for the three types of screw in Fig. 37.

This formula for thrust so far as its main factors are concerned rests upon a reasonable theoretical basis. The normal pressure experienced by any blade inclined at a small angle to its line of motion, is known by theory and experiment to vary directly as this angle, the (velocity)^2 and its area. If any element of a screw advancing axially at speed v_1 be revolving at a circumferential speed given by n_0 so that it develops no thrust, the angle of the element to the plane of rotation is measured by $\left(\frac{v_1}{n_0}\right)$. At any other circumferential speed (n) when developing thrust T per unit area the inclination of the path of the element is given by $\frac{v}{n}$. T is therefore measured by

$$n^2\left(\frac{v_1}{n_0}-\frac{v_1}{n}\right)=\frac{v_1}{n_0}\times\frac{n-n_0}{n}\times n^2.$$

The first of these three terms is the effective pitch, the second is the slip ratio, and n is the product $N\times D$.

Hence T per unit area = "a" N^2D^2sp.

Since area for given type of blade varies as D^2 we may write

$$T=\text{“}a\text{”}\ N^2D^4sp,$$

which can be put in the form

$$\frac{T}{D^2V_1^2}=\frac{\text{“}a\text{”}}{p}\ \frac{s}{(1-s)^2}.$$

This is of the same general form as equation 7, the unknown term "a" being there replaced by the factors

$$1{\cdot}02B(p+21)(1-{\cdot}08s).$$

These two latter terms take account of the experimental variation

of the thrust factor with pitch and slip for any fixed disc area ratio of blade.

It will be seen that the "B" value for any screw is a direct measure of the thrust which it will develop, and the experiments show that with given blade area, diameter, and slip the "B" value for the four-bladed elliptical screw is roughly 12 per cent. higher, and for the three-bladed wide-tipped screw is roughly 7 per cent. higher than that for the three-bladed elliptical.

These higher thrust values are accompanied by slight losses in efficiency, the wide-tipped screw being 3 per cent. and the four-bladed screw 2 per cent. worse than the three-bladed elliptical. In practice the three-bladed screw would have a greater root thickness than the four-bladed screw of the same area and diameter, and it will be seen from the section on root thickness that this modification would have caused a little loss in efficiency. The efficiency advantage of the three-bladed screw over the four-bladed screw is therefore more apparent than real. Moreover, the four-bladed screw developing the same horse-power as a three-bladed screw of the same area, would work at different pitch or slip, and the effect of this upon efficiency may, and often does, more than balance any small disadvantage the screw has from number of blades. Experiments were made by Mr. W. G. Walker with a single-screw vessel 55 feet in length, varying the number of propeller blades from two to four, and from three to six, keeping the total area the same. The propellers were approximately ·4 disc area ratio and 1·66 pitch ratio. The number of blades had practically no effect upon the efficiency, and very little upon the revolutions at any reasonable speed.

The three-bladed wide-tipped pattern used in the experiments was narrower at the root than is usual in practice, and the thickness at root relative to width at root was in consequence relatively greater than it should be in comparison with the three-bladed elliptical. The easing of the angles of these root sections would modify the efficiency, not to its detriment; and the difference in efficiency found by Froude between this and the elliptical type of blade may be regarded as an extreme value.

Before leaving this question of thrust values it is desirable to draw attention to the very small increase of thrust that accrues from a comparatively large increase in blade area when the disc area ratio has reached a moderate value. This is true for all types of blade and all pitch ratios, and it is clear that, where circumstances permit, it is better to use a large diameter of small disc area ratio rather than a smaller diameter and wider blades.

§ 74. Efficiency.—The efficiencies obtained with the various screws are given in the form of a series of curves in Fig. 39. These are drawn to a base of x value, each for a round number pitch ratio. They are correct for the three-bladed elliptical type of screws having a standard disc area ratio of ·45. To correct them for another type, a uniform deduction must be made of

·024 for the wide-tip three-blade ;
·0125 for the four-blade elliptical.

A correction is also necessary for disc area ratio, and the correction curves for this are given in Fig. 37 to a base of disc area ratio.

All the efficiency curves have a common characteristic shape, having a maximum value at a slip of about 20 per cent. ($x=1{\cdot}27$) for large, and 25 per cent. ($x=1{\cdot}35$) for small pitch ratios. The efficiency falls away slowly for slips above these values, and rapidly at slips below 15 per cent. ($x=1{\cdot}19$).

The curves for high pitch ratios attain greater maximum values, but fall off in value more rapidly at high slips, than do the curves for low pitch ratios, so that the advantage of a high pitch ratio becomes smaller the higher the slip the screw is worked at, and disappears altogether for slips above about 40 per cent.

The effect of large disc area ratios is shown by the correction curves in Fig. 37, but can also be seen in Fig. 38, which shows the maximum possible efficiency for definite disc area ratios, when pitch ratio is varied. These curves show a maximum value for small disc area ratios of ·745. The value obtained by Mr. Taylor*

* Mr. T. B. Abell's analysis of Taylor's experiments shows practically the same efficiency as that obtained by Froude, but for the low pitch ratios Taylor's experiments show slightly less efficiency at high slips and more at low slips. The difference is, however, not more than 1 or 2 per cent.

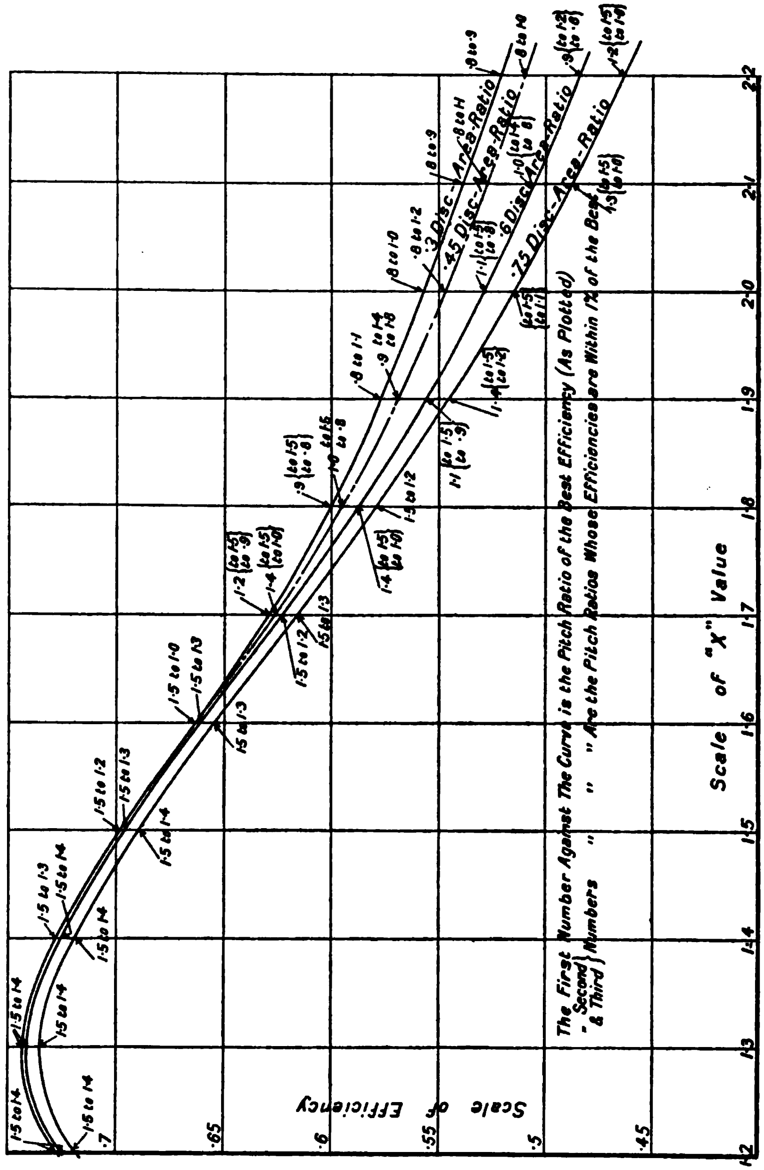


FIG. 38.—Maximum possible Efficiency at any fixed Disc Area Ratio.

for a screw of the same disc area ratio and pitch ratio but of larger diameter (1·33 feet), is ·77, or 3 per cent. greater, probably due to the effect of blade thickness, or the larger size of screw used.

These results show that there is for any screw a fairly large range of slip at which the efficiency remains constant at its maximum value. The dimensions of any screw should be so arranged that the slip falls about the *higher* limit of this slip range, or as near to this on the higher side as possible. The lower side of this slip range should be avoided, as, if through any cause the screw works at a lower slip than the estimate, the efficiency loss may be of considerable importance, owing to the rapid decrease in efficiency with slip at this end of the curve. The general flatness of the efficiency curves at their maximum value, and the small change resulting from variation of pitch ratio when this exceeds 1·2 enables the propeller dimensions to be varied over a fairly wide range without any material loss of efficiency, provided they work in this region of slip and pitch. This is the case for a large number of vessels of comparatively low speed and is borne out in practice.

§ 75. Use of Model Screw Data for Estimating Purposes.—To facilitate design calculation or steam trial analysis the formula for thrust in § 73 is better expressed in terms of thrust horse-power (H), speed of screw through wake water in knots (V_1), revolutions in hundreds per minute (N), diameter in feet (D) :—

Thus

$$\frac{T}{D^2 V_1^2} = \frac{T V_1}{D^2 V_1^3} = \frac{H\left(\frac{33{,}000}{100}\right)}{D^2 V_1^3\,(1{\cdot}0133)^3} \text{ where } V_1 \text{ is now in knots,}$$

whence

$$\frac{H}{D^2 V_1^3} \times \frac{p}{B(p+21)} = {\cdot}003216\,\frac{s(1-{\cdot}08\,s)}{(1-s)^2} = Y.$$

Writing

$$X = \frac{NpD}{V_1} = \frac{1{\cdot}0133}{1-s}.$$

It will be seen that these two terms Y and X involve only one variable—slip—and the former can be plotted to a base of the latter. Such a curve is given in Fig. 39, together with curves of efficiency for several pitch ratios all for disc area ratio ·45. This X—Y curve serves very well for the analysis of steam trial data,

and an example is given in Table 33 in order to show its use and the better to define the terms and units used.

For propeller design the data is not very convenient in this form. Both X and Y contain diameter and pitch ratio, and both these are usually unknown. The propeller has to be designed to develop a given power at fixed revolution and fixed ship speed, and the work can be done better when the data is arranged so that one term does not contain diameter. Since

$$Y=\frac{H}{D^2 V_1{}^3}\,\frac{p}{B(p+21)}$$

and

$$X=\frac{NpD}{V_1}$$

$$X^2\times Y=\frac{N^2H}{V_1{}^5}\times\frac{p^3}{B(p+21)}\text{ (a term independent of diameter).}$$

Values of $X^2\times Y$ are plotted to a base of X in Fig. 40. For any particular ship whose NH and V_1 are known the practice is to assume three or four pitch ratios and two disc area ratios (or "B" values) and to find the values of $X^2\times Y$ for all of these. From the curve of $X^2\times Y$ the values of X $\left(i.e., \frac{NpD}{V_1}\right)$ corresponding to these X^2Y values are obtained. Since N and V_1 are known the diameters can at once be found. A curve of diameter for each disc area ratio can then be plotted to a base of pitch ratio. The efficiencies are also known, being those given by the efficiency curves for the chosen "p" values at the proper "X" values. Some slight correction may be required for disc area ratio or number of blades as already mentioned, but when corrected these values may be plotted in the same way as the diameters. A diameter and "p" value can then be chosen from these curves to give a reasonable clearance between blade tip and ship with the best possible efficiency. An example which will make this clearer is given on page 175.

When starting these calculations the following facts must be borne in mind :—

(a) The data given is for screws in open water. The speed V_1

is for the screw behind the ship, and must be taken as speed of screw through the wake water, and not as speed of ship. Some wake factors are given in Tables 29, 30, 35 A, B, C, D, to enable V_1 for the screw to be obtained from the ship's speed. But the whole section dealing with wake should be read in order that a sound estimate of this factor in any particular case may be made from the data given.

(*b*) The driving face pitch of the ship screws, as already explained, is $\frac{1}{1{\cdot}02}$ times that obtained from these calculations. This figure is based upon Admiralty trials before 1906, when pitch ratios of 1·0 to 1·2 combined with moderate disc area ratios were general. The author has found that for turbine vessels having screws of smaller pitch ratios (·85 to ·95), and larger disc area ratios, a somewhat higher denominator (1·04) gives better agreement between estimate and trial results.

When the driving face is cut away slightly near the *leading edge*, as is done in some propellers, the effect is to slightly increase the true pitch, and the factor then becomes somewhat less than $\frac{1}{1{\cdot}02}$; and, in a similar way, cutting away the driving face at the *trailing edge* increases the factor to $\frac{1}{1{\cdot}01}$ or less.

For *model* screws Mr. Froude's results give an average figure of $\frac{1}{1{\cdot}095}$. Mr. Taylor's results as analysed by Mr. Abell, show a factor varying from $\frac{1}{1{\cdot}05}$ for large disc areas, to $\frac{1}{1{\cdot}10}$ for small disc areas of 1·2 pitch ratio, and $\frac{1}{1{\cdot}15}$ for small disc areas of ·8 pitch ratio. Herr Gebers' experiments show a factor of $\frac{1}{1{\cdot}085}$, his screws being of unity pitch ratio, and fairly large disc area ratio. In the absence of definite and complete results from steam trials these figures may be regarded as limiting values to the coefficient showing how it probably varies, and not as figures to be used for full-sized screws.

(*c*) It must be understood that these efficiencies are for clean, smooth blades in which the skin friction is normal. The effect of a rough surface is shown by the following experiment. A 6 feet diameter propeller was roughened so that the surface was equivalent to coarse sand. It was used to propel an 18·5 ton vessel of about 60 feet length. To get the same speed as was obtained with the same propeller, but with a smooth surface, it was necessary to increase the power from 12 to 20 per cent. and the revolutions 8 per cent. Experiments with a flat blade in air at small angles showed that varying the surface from "varnished mahogany" to "rubber surface fabric" made practically no difference to the thrust or lift, but increased the resistance from 18 per cent. at small angles to 7 per cent. at an inclination of 12 degrees; very few screws work with their blades inclined to their direction of motion at an angle greater than 12 degrees.

(*d*) For a three-bladed propeller of radius r, having a blade width ratio of, say, ·4, the developed area assuming there was no boss

$$= 3\left[\frac{\pi}{4}\times(\cdot 4)r^2\right] = \cdot 3\pi r^2$$

and the disc area ratio

$$= \frac{\cdot 3\pi r^2}{\pi r^2} = \cdot 3.$$

The disc area ratios used by Mr. Froude in plotting his results were obtained in this way, and this fact must be borne in mind in using the data.

If the developed outline of the blades be produced to the shaft centre and the boss line be drawn on the diagram, it will be found that the proportion of the whole outline cut off by the boss varies from 20 per cent. for Admiralty pattern screws of moderate revolutions, to about 15 per cent. for wide-tipped turbine-driven screws cast on the boss. The actual figure can easily be obtained in any particular case by measurement from similar screws. When using Froude's data for obtaining the dimensions of propellers to satisfy given conditions, some such discount must be put upon the area obtained by calculation in order to obtain the correct

developed area of the blade outside of the boss line. Similarly, when analysing trial results, the actual area of the blades should be increased by a similar percentage (which can be obtained with fair accuracy from the plan of the propeller), and this increased value should be used for obtaining the disc area ratio from which the "*B*" value is obtained. Great accuracy is not required in obtaining these disc area ratios, as the thrust is not very sensitive to area unless the blades are very narrow or cavitation is present.

Screw Dimensions by Means of the X^2—Y Curve.—It is required to find suitable dimensions for four screws to run at 315 revolutions per minute, to develop a speed of 20 knots in a turbine-driven vessel whose effective horse-power is 10,000.

Allow for bilge keels 2 per cent.; for bossings, 3 per cent.; air resistance (fine weather), 2 per cent.—total, 7 per cent.

Hull efficiency assumed, ·98.

$$\left(\begin{array}{c}\text{T.H.P. to be delivered by each}\\ \text{of the four screws}\end{array}\right)=\frac{10{,}000\times 1{\cdot}07}{{\cdot}98\times 4}=2{,}730=H.$$

With a clearance of about 14 inches the wake factor for this vessel may be taken as ·16.

Speed through wake water, $V_1=\frac{20}{1{\cdot}16}=17{\cdot}24$ knots.

Thrust on each screw in lbs. $=\frac{2{,}730\times 33{,}000}{17{\cdot}24\times 101{\cdot}3}=51{,}540$ lbs.

The screws are well buried and a cavitation pressure of 13 lbs. per square inch may be assumed.

Minimum blade area $=\frac{51{,}540}{13\times 144}=27{\cdot}55$ square feet per screw.

$$\frac{N^2H}{V_1^5}=\frac{(3{\cdot}15)^2\times 2{,}730}{(17{\cdot}24)^5}={\cdot}0178$$

$$\frac{V_1}{N}=5{\cdot}47$$

The results are plotted in Fig. 41. Other disc area ratios can be used in the calculations if desired. The screw should be chosen

from the diagram to the right of the cavitation line, with as large a pitch ratio as possible, consistent with good efficiency.

Table 25.

Calculation for Three-bladed Screws.

Disc Area Ratio assumed :	·5				·6			
"B" Value for moderately Wide-tipped Screws from Fig. 37 .	·11				·113			
$\frac{N^2H}{BV_1^5}$	·1619				·1576			
p assumed . . .	·8	·9	1·0	1·1	·8	·9	1·0	1·1
$\frac{p^3}{p+21}$	·0235	·0333	·0455	·0603	·0235	·0333	·0455	·0603
$\frac{p^3}{p+21} \times \frac{N^2H}{BV_1^5} = X^2Y =$.	·0038	·0054	·0074	·0098	·0037	·0052	·0072	·0095
Hence from Fig. 40, $X=$	1·435	1·52	1·602	1·685	1·43	1·511	1·594	1·676
$D = X \times \frac{V_1}{Np} =$ . . .	9·8	9·23	8·76	8·38	9·75	9·18	8·72	8·33
Face pitch $\frac{pD}{1{\cdot}03}$ . . .	7·61	8·07	8·51	8·95	7·57	8·03	8·47	8·9
Developed area of 3 blades in square feet $\cdot 85 \times \frac{\pi}{4} D^2$ $\times$ disc area ratio $=$.	32·0	28·4	25·6	23·4	38·2	33·9	30·6	27·9
Efficiency for above X values at corresponding p values, from Fig. 39 .	·65	·655	·654	·628	·65	·656	·655	·631
Correction for disc area ratio, from Fig. 37 .	−·008	−·006	−·005	−·003	−·028	−·023	−·018	−·013
Corrected screw efficiency η	·642	·649	·649	·625	·622	·633	·637	·618
S.H.P. required at the propeller $= \frac{H}{\eta} =$ . .	4,260	4,210	4,210	4,370	4,390	4,310	4,290	4,420

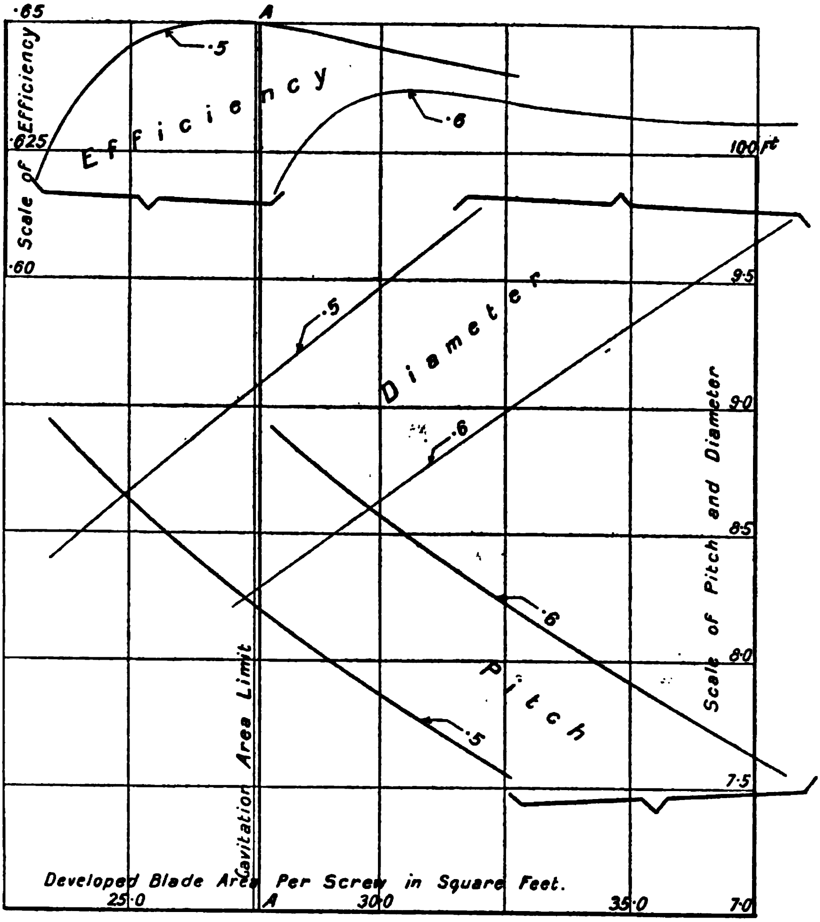


FIG. 41.—Typical Diagram of Screw Dimensions and Efficiency.

Good dimensions would be—diameter, 8·75 feet; pitch, 8·45 feet; developed area, 29·5 square feet per screw; disc area ratio, ·575; efficiency of screw, ·63. These dimensions are obtained from the above by interpolation.

CHAPTER XX

PROPELLER BLADES

§ 76. Pitch Ratio.—It will be seen from the theoretical formula in § 72 that the thrust produced by a screw at any slip will be directly proportional to the pitch if the revolutions remain the same. The same fact may be stated in another way. If a ship's speed and power are to remain the same, and the screw pitch is altered without affecting the diameter or blade area, then as the pitch is increased the revolutions will decrease, and the slip will increase. The effect of the change upon the efficiency would depend upon the initial pitch ratio and slip. For ordinary screws of pitch ratios 1·0 to 1·3, working at slips of 20 per cent. and above, there would be little change, the loss due to higher slip and the gain due to higher pitch ratio being practically equal. But at slips exceeding 30 per cent. there is a fair loss of efficiency with increase of slip, but practically no advantage due to using higher pitch ratios than 1·1. Increase in pitch ratio would therefore, in this case, mean loss of efficiency.

All blades, and particularly wide ones, of pitch ratio ·8 to 1·0 gain considerably in efficiency by increase of pitch ratio, and since they do not reach their best efficiency until the slip is 25 per cent. it follows that it is a distinct advantage to keep the pitch as high as possible, even though it means a fairly high slip ratio. Thus a propeller of ·75 disc area ratio of 1·0 pitch ratio, has as good efficiency at 38 per cent. slip, as a propeller of the same area and ·8 pitch ratio has at 25 per cent. slip, and at any slip lower than 38 per cent. the 1·0 pitch ratio is always better, its advantage being over 10 per cent. at 25 to 30 per cent. slip.

This disadvantage of small pitch ratio is due to two main causes :—First, small pitch ratios involve small slip angles, unless

the slip ratio is large, and no blade works well at small angles. Thus a pitch ratio of ·8 has slip angles of 2°·09 and 4°·98 at slip ratios of 15 per cent. and 35 per cent. A slip angle of 2°·9 is required for a reasonable efficiency, which will remain fairly good until the slip angle exceeds about 6°·0. Secondly, small pitch ratios involve small pitch angles, which give less clearance or smaller gaps between consecutive blades, a point dealt with later (see page 182).

§ 77. Rake and Skew-back.—Experiments with model screws show that efficiency and thrust are not affected by raking the blades either forward or aft up to 15 degrees. Rake aft may be of advantage in twin-screw ships in obtaining greater clearance of propeller tip from hull, and in single screw ships gives a greater distance between the body post and the leading edge of the blades, a desirable thing, particularly for vessels having full after lines. It has the disadvantage, however, of throwing greater stress upon the propeller blades due to the centrifugal forces. If the thickness of the root sections of the blades have to be increased to resist the additional strain brought about by the rake, the efficiency will not be improved, and in high revolution propellers will fall slightly.

No material advantage is gained in the ordinary vessel by bending the blade in the transverse plane. For vessels working in waters containing a lot of weed, etc., a certain amount of skew-back may be found advantageous, as it helps the blade tips to clear themselves of the weed.

Developed Outline.—The relative merits of the elliptical and wide-tipped pattern blades have already been considered (§§ 73 and 74). The thrusts obtained by Taylor with screws of a different type of outline (see Fig. 42) were slightly lower than those for Froude's elliptical blades, the efficiency being about the same for both. The root thickness of Taylor's blades was slightly less than that of the others, which may account for this difference, and generally speaking the two *outlines* may be said to give approximately the same results.

The blade outline should be well rounded at the corners, as such shapes as the "very wide tipped" in Fig. 42 tend to produce

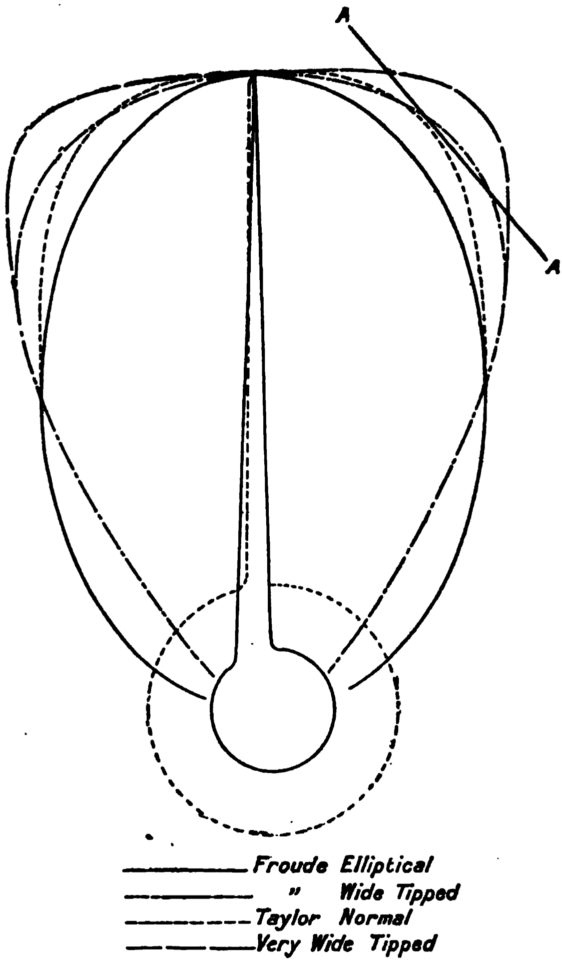


Fig. 42.—Outlines of Blades.

vibration, particularly if the clearance between the blade tips and the hull plating is less than about 12 inches.

Such corners if they exist at the leading edge of the blade are a source of weakness, and are liable to bend across a line as *AA*. The propeller thrust and suction per square foot are both greatest

at a point approximately ·092 of the diameter from the tip, and at any radius is greatest near the leading edge, so that with a corner there is a tendency to local concentration of force at this part. By rounding the edge at the tip the thrust is distributed better over the blade, the length of the section on such a line as *AA* (cutting off a constant area) is much greater, and the stress on it less.

Disc Area Ratio.—Large disc area ratio may be obtained by

(*a*) Increasing the number of the blades;
(*b*) Increasing the area of individual blades.

All model experiments show that, with a fixed number of blades of moderate width, at any slip the variation of efficiency with disc area ratio is negligible. But for disc area ratios of ·55 and above there is a loss of efficiency which becomes more important the smaller the pitch ratio (see Fig. 37). Thus, for propellers of ·8 pitch ratio the maximum possible efficiencies at ·4 and ·8 disc area ratios are ·66 and ·56 respectively. With engines running at high revolutions, the tendency is towards small pitch ratios of large disc area ratio, and the propulsive efficiency necessarily suffers through the low screw efficiency.

The reader's attention is directed to § 73, where the relative merits of three and four bladed screws are considered. It is there shown:—

(1) That somewhat more thrust is developed by four blades than by three, the diameter and total area being the same, there being only a very slight difference in efficiency in favour of three blades.

(2) That above a disc area ratio of about ·55 large increase in area produces but small increase in thrust. For this reason a wide blade propeller is stronger than a narrow one of the same root thickness; there is very little increase in thrust and a considerable increase in root area to stand the strain.

Large area ratios increase the backing power of the screws, and may be required for this reason. But, unless the propeller is

working near the cavitation limit and the diameter is restricted, large disc area ratios have no other propulsive advantage.

The effect of wide blades upon efficiency is given by Mr. Froude as largely independent of slip but varying with pitch ratio, and must therefore be due to variations made in the design of the blades when varying the widths. Increase of width was accompanied by

(1) Smaller ratios of $\dfrac{\text{blade thickness}}{\text{blade width}}$;

(2) Larger ratios of $\dfrac{\text{blade width}}{\text{propeller radius}} = \dfrac{b}{r}$;

(3) Smaller ratio of $\dfrac{\text{blade width}}{\text{gap}}$.

By "gap" is meant the normal distance between the spiral surfaces of consecutive blades. A propeller of n blades having a pitch angle θ at any radius r, has a normal gap between the blades at this radius equal to $\dfrac{2\pi r \sin\theta}{n}$ and the ratio $\dfrac{\text{blade width}}{\text{gap}}$ at the radius r equals $\dfrac{nb}{2\pi r \sin\theta}$.

The effect of $\dfrac{\text{blade width}}{\text{gap}}$ has been tested with a number of aerofoil blades (*i.e.*, with biplanes), and by means of Mallock's method the results can be used to give an indication of the effect of this ratio on a screw. If the pitch and slip be maintained the same, and $\dfrac{\text{blade width}}{\text{gap}}$ is varied from $\frac{5}{3}$ to $\frac{5}{6}$ by reduction of blade width, the thrust per square foot will be increased 25 per cent. so far as gap alone affects it. In the latter case there is only half the area of blade, but the thrust is

$$\tfrac{1}{2} \times 1{\cdot}25 = \tfrac{5}{8}\text{ths that of the wider blade.}$$

This increase in thrust per square foot as $\dfrac{\text{blade width}}{\text{gap}}$ is decreased is accompanied by an increase in efficiency of about 2 per cent. at high pitch ratios (1·2) and 7 per cent. at low pitch ratios (·8).

But increase in blade width gives a larger ratio of $\dfrac{\text{mean blade width}}{\text{propeller radius}}$

and tests with blades in both air and water show that the thrust and efficiency depend upon this ratio. This can be seen from the following table :—

TABLE 26.

Thrust per Unit Area and $\frac{\text{Thrust}}{\text{Resistance}}$ for Blades having different Ratios of Width to Length.

Ratio $\frac{b}{r}$.	Flat Blades in Water.		Rounded Back Blades in Air.	
	Thrust per Unit Area.	$\frac{\text{Thrust}}{\text{Resistance}}$.	Thrust per Unit Area.	$\frac{\text{Thrust}}{\text{Resistance}}$.
2·0	·104	5·2		
1·0	·165	6·2		
·5	·24	8·3		
·33	—	—	·28	10·0
·25	—	—	·32	11·5
·20	—	—	·325	12·9
·167	—	—	·33	14·0

The thrusts in columns 2 and 4 are in the same units : b is measured in the direction of motion and r across it.

Reducing the ratio $\frac{b}{r}$ from 1·0 to ·5 gives an increase in thrust per square foot of $\frac{\cdot 24}{\cdot 165}=1\cdot 45$. The smaller blade therefore gives $\frac{1\cdot 45}{2}=\cdot 725$, the thrust of the larger blade of double the area, and there is a considerable increase in $\frac{\text{thrust}}{\text{resistance}}$ and therefore efficiency with the narrow width.

The definite figures given above are not intended to be anything but a guide to what happens with a screw, but taking them as they stand, the thrust for a narrow-bladed screw and one of twice the width of blade ($\frac{b}{r}=1\cdot 0$) are in the ratio :—

$$\underbrace{\tfrac{5}{8}}_{\frac{\text{blade width}}{\text{gap}}\text{ effect}} \times \underbrace{1\cdot 45}_{\frac{b}{r}\text{ effect.}} = \cdot 9$$

Froude's "B" values for the corresponding blade widths are in the ratio ·91.

It will be seen that whatever tends to decrease the value of $\frac{2\pi r \sin\theta}{nb}$ tends to decrease the efficiency and the thrust per square foot at any slip of the propeller. Wide blades and small pitches (except at high slips) each give poor efficiencies, without considering the effect of the blades upon each other, and when this interference effect is taken into account they become still more undesirable. Small pitch ratios mean small pitch angles (θ), and when combined with wide blades (or large b values) can never give good results.

§ 78. Thickness and Shape of Blade Section.—Some experiments have been made by Mr. Taylor to test the effect of blade thickness.

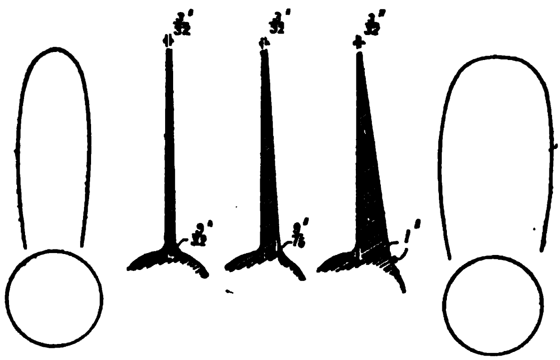


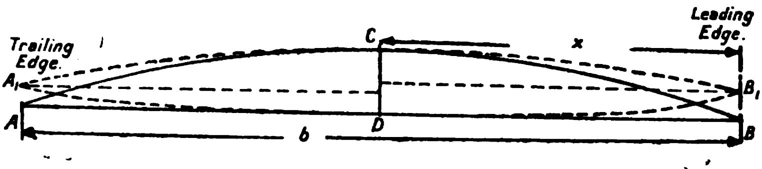
FIG. 43.

His screws were 16 inches in diameter, varying in pitch ratio from ·6 to 1·2, having developed blade areas of 39·3 and 62·9 square inches in three blades (mean width of blade 2·5 and 3·73 inches respectively). Three root thicknesses were tried—viz., $\frac{9}{32}$, $\frac{9}{16}$, and 1 inch (see Fig. 43). It was found that

(*a*) The effective pitch of the screw increased with the thickness, so that fewer revolutions would be required with the thick blades to produce the same thrust as the thin ones, the difference in revolutions being about 12 per cent. for the extreme variation of thickness tried.

(*b*) Provided the ratio $\frac{\text{maximum thickness at half radius}}{\text{mean width of blade}}$ did not exceed ·045 the efficiency remained practically the same when thrust, pitch, and area were the same. When this ratio exceeded about ·07 the efficiency fell off badly.

Experiments with blades in air support the above conclusions. These also show an increase in effective angle (or slip) when the thickness is increased. The efficiency of such blades at the same thrust improves very slightly for small slips till the ratio $\frac{CD}{AB}$ in Fig. 44 equals about ·06, after which it decreases slowly. But for slip angles much in excess of 12 degrees (effective) the efficiency



ADBC: Section of Screw 139.
A_1DBC: Section of Screw 143.

FIG. 44.

breaks down rapidly when the thickness exceeds that given by the above ratio (·06).

It must be remembered that the above results only hold provided proper screw action is taking place; the differences due to the thickness *may* be smoothed out as cavitation sets in.

The **effect of shape of blade section** has been tested on a set of screws of rather fine pitch ratio (·7875 on the face), the blade width ratio being ·6.

A screw is usually required to develop a given thrust, and comparing the results at constant thrust the following conclusions are arrived at:—

(1) Setting back the driving face at the leading edge from B to B_1 (Fig. 44) has little effect upon the revolutions, and gives slightly better efficiency at all moderate slips. But with a propeller of higher pitch ratio=1·2 on the face, there was a serious

falling off of efficiency at high slips, the loss being 7 per cent. at 14 per cent. slip.

(2) Setting back the trailing edge from A to A_1 requires higher revolutions to produce the same thrust, but has little or no effect upon the efficiency except at very low thrusts, when the results are rather indeterminate, but seem to indicate that a little set-back may do good but too much will cause a serious loss of efficiency. The increase in revolutions passing from section 139 to 143 is 10 to 15 per cent., and varies with the ratio $\frac{AA_1}{AB}$ for intermediate sections.

Experiments with blades in air support the above conclusion as regards increase in revolutions to obtain the same thrust when the driving face is cut back at the trailing edge. Such experiments also show a slight gain in efficiency at small thrusts with small set-back (up to $\frac{AA_1}{AB} = \cdot 02$) and a loss of efficiency at high thrusts, which becomes serious if the set-back exceeds a ratio of $\frac{AA_1}{AB} = \cdot 05$.

Position of Maximum Thickness in Section of Blade.—The shape of the section of a blade is usually symmetrical about the axial line. This is adopted mainly on the score of strength going ahead and astern. There are no published results of tests with propellers having the maximum thickness elsewhere than in the middle of the section, which would show if anything can be gained by keeping the maximum thickness near the leading or trailing edge. Some guidance can be found in tests of flat blades in air, and the following paragraphs give the general conclusions to which such tests lead one :—

(*a*) There is a fair gain in efficiency by moving the maximum thickness towards the leading edge, provided the ratio $\frac{x}{b}$ is greater than $\cdot 3$ (see Fig. 44). If $\frac{x}{b}$ is made smaller than $\cdot 3$ the efficiency drops, particularly at very high slips.

(*b*) The thrust is improved by making $\frac{x}{b}$ equal to ·38, but for small values of this ratio the thrust falls off badly at the higher slips.

It appears, therefore, that when going ahead more work can be put into the screw with a fair gain in efficiency by keeping the point C a little to the leading edge, provided that $\frac{x}{b}$ is not made less than ·38.

(*c*) Thickening the section on the back towards the trailing edge has very little effect upon either the thrust or efficiency, within the limits of the experiments. These are shown by sections 1 and 3, Fig. 45.

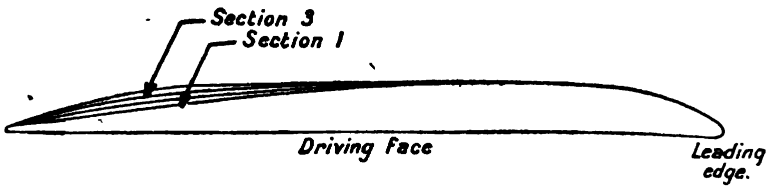


Fig. 45.

Hollowness of Driving Face.—It is not uncommon for screws of moderate width to have increasing pitch from the leading to the trailing edge, *i.e.*, to be hollow on the driving face. Thornycroft's model screws show slightly better efficiency with the pitch of the driving face varied in this way, but the experimental results are not given in sufficient detail to allow of a comparison of thrusts, and in any case the blades have other special characteristics besides that being discussed.

Experiments with flat blades show that the hollow face has little or no advantage in efficiency, but develops a given thrust at a slightly smaller angle than the flat face (measured on the chord *AB* in Fig. 46 in both cases). At slip angles exceeding 12 degrees the hollow face has a considerably better thrust coefficient than the other.

A hollow face, therefore, would have no advantage at the root of a screw blade where the slip angle is small, but could be worked

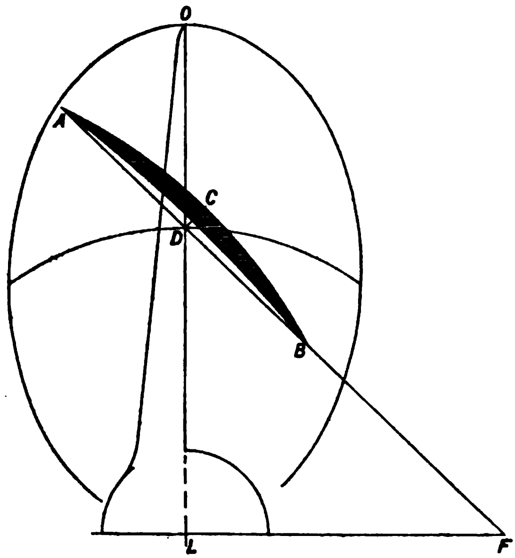


FIG. 46.

with advantage to the thrust over the central portion of the blade. The screw would then develop a given power with slightly less revolutions than the flat face.

CHAPTER XXI

HULL EFFICIENCY, WAKE AND THRUST DEDUCTION

§ 79.—The second term in the net efficiency of propulsion given on page 162 is the hull efficiency, *i.e.*, the ratio

$$\frac{\text{work done in towing the ship at any speed}}{\text{actual work done by the screw in propelling the ship at that speed.}}$$

It has already been shown that this factor may be written in the form

$$h=(1+w)(1-t).$$

One of these terms depends upon the wake velocity, the other upon the extent to which the action of the screw augments the resistance of the ship. The two terms are not independent of each other, and any condition which tends to increase the wake factor w generally tends to increase the thrust deduction factor t, so that h remains fairly constant, varying as a rule between ·95 and 1·0. For twin-screw vessels of moderate prismatic coefficients at speeds above that given by

$$V \text{ (knots)} = \sqrt{L \text{ (feet)}}.$$

provided that the propeller clearance is good ·96 may be taken, and for lower speeds ·97 to ·98. These figures are for vessels in which eddy-making at the stern is absent; with after-body prismatic coefficients much in excess of ·7 they become smaller.

Hull efficiency *as a whole* is therefore not of great interest to the designer. But the separate factors which go to its make-up, owing to their influence in other directions are always important. A proper valuation of w is necessary in order that the velocity of the screw *through the water* may be obtained. If the thrust on the screw is approaching the cavitation limit, then the augmenta-

tion of the resistance becomes important, and anything which will reduce this, means smaller thrusts for given speed and power developed.

Wake.—The natural stream line flow of the water past the ship, produces around the after-body a forward velocity which differs in magnitude at different parts of the stern, but which *at any point* is a fairly constant percentage of the ship's velocity. The wake velocity due to this is always positive, *i.e.*, in the direction of motion, but is smaller the further forward the screws are placed. It has its greatest value near the ship, and smallest outboard at the keel level.

Two or three other causes, however, are at work, their effect varying according to the type of ship. The surface of every ship being frictional, is bound to produce a forward velocity in the water with which it may come into contact. This forward velocity is transmitted in a minor degree to the surrounding particles. Hence the "frictional belt" tends to increase in thickness continuously towards the stern, losing at the same time any definite line of demarcation between it and the surrounding water.

But with very full lines, stream line flow becomes impossible near the body post, and the vessel tends to drag some water along at a velocity equal to its own. This is particularly the case in the region of the water line, where full lines may be required for stability or other reasons.

At high speeds, waves are formed by the ship, and the particles of water constituting any wave have an orbital velocity. At the crest of a wave this orbital velocity is forward, *i.e.*, in the direction of motion; in the hollow of the wave the particles move backwards. It follows that the wake effect of this orbital velocity will depend upon the position of the wave crests and hollows at the stern of the ship. A screw working directly under a wave crest will have a large forward wake, but if it is placed near a wave hollow, the wave effect will tend to cancel the frictional belt and stream line effect and give a negative wake. Thus all destroyers at high speeds travel with the stern in a small wave hollow, and

the wake factor for these is sometimes negative, and seldom more than 2 per cent. positive.

These causes together produce a wake of varying intensity at different points around the stern, and the experiments of Calvert have shown the extent of this variation. He measured by means

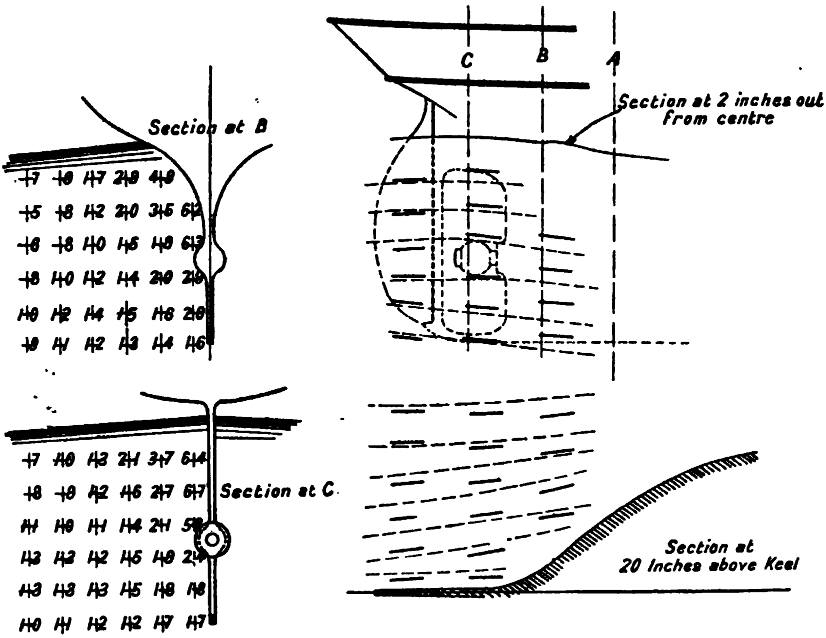


FIG. 47.—Calvert's Wake Experiments.
The figures on the sections give the forward movement (or positive wake) of the water at any point, as a percentage of the forward movement of the boat.

of Pitot tubes the velocity at various points around the stern of a ship, and Fig. 47 shows the wake values he obtained.

The designer is mainly concerned with that part of the wake in which the screw works, and the wake factors determined by screw experiments, or by the analysis of steam trials, give the mean wake over this area, and are used as already explained. In Tables 35 A, B, C D, is given a list of ships and the wake factors (w) obtained generally by model experiments. The speed V_1 of the screw through the water is given by

$$V_1 = \frac{V}{1+w}.$$

These values apply generally to screws with good clearances between hull and tip of blades, such as are usually worked in twin-screw ships, viz., from 1 to 2 feet, not having any obstruction in front which might materially affect the flow of water.

§ 80.—The following additional remarks are based upon Froude's and Luke's experiments :—

(1) The wake factors are practically independent of the screw pitch, and number or area of blades, as theory would lead one to expect.

(2) Wake decreases very slightly with speed for all practical speeds.

(3) With a single screw, diminishing the diameter increases the wake value, but decreases the thrust deduction value as well, the hull efficiency increasing somewhat with smaller diameters. It must be remembered that this result could not be obtained in a ship having very full after lines owing to the dead water effect. Decrease of diameter in any ship would be accompanied by higher slip, and probably lower screw efficiency, which discounts some, if not all, of this apparent gain in hull efficiency.

(4) For low-speed vessels working with a slip of about 22 per cent., there is the danger of getting on the low slip side of the efficiency hump. This can be avoided either by designing the screw for a slightly higher slip than that which gives the maximum possible efficiency, or by using a wake factor slightly lower than the tables give. The loss in efficiency by doing this is not important, and a good result is assured.

(5) The fore-body of the ship has practically no influence upon hull efficiency or wake.

(6) Increase of the prismatic coefficient of the after-body, keeping the screw diameter and tip clearance the same, and with the same fore and aft position of the screw, leads to lower hull efficiency values, the loss becoming more marked at those speeds at which heavy wave-making occurs. This loss is due partly to slightly lower wake factors, but more to higher thrust deduction values with the fuller form. If the screws be kept on the same

centre line as the stern is filled out, and are shifted further aft so as to maintain the same clearance of the blade tips, the hull efficiency does not alter to any great extent. This is an important matter, and must be borne in mind when the stern is filled out to obtain lower towing resistance.

(7) Variation of clearance between the tip of the propeller and the ship has a considerable effect upon the wake and thrust deduction. This effect is generally the same whether obtained by varying the spread, fore and aft position, or diameter of the screws. Small clearances give higher wake factors and higher thrust deduction values. The greater the clearance becomes, the nearer to unity does the hull efficiency approach. So that a low hull efficiency can be improved by giving the screw a greater clearance, but, unless a smaller wake factor is desired, or a smaller thrust deduction, there is nothing to be gained by increased clearance if the hull efficiency is already close to unity (see effect upon cavitation).

(8) The smaller the wake experienced by a screw, the smaller is the real slip at any ship speed and revolutions, and the higher can the revolutions be worked before reaching the cavitation limit. If this smaller wake factor is obtained by large clearance, the presence of the hull has less effect upon the thrust exerted by the propeller tips *as they pass the hull*—*i.e.*, the pressure on each blade remains more uniform throughout its revolution. This is important in high-speed ships, and is dealt with in the section on Cavitation.

(9) The lower a screw can be kept relative to the hull, the smaller will both thrust deduction and wake become, and the *nearer to unity* will the hull efficiency approach. A low position of the screws also has the advantage that the screws have less chance of breaking the water surface when the vessel is at a light draft or pitching.

This effect in a more or less exaggerated form can be seen from the following table, which gives the results of trials of two self-propelled barges of ·9 block coefficient :—

TABLE 27.

Trials of U.S. Navy Oil Barges Nos. 2 and 3.

Number of Column :—		2	3	4	5	6
Propeller dimensions	diameter (feet)	5·5	6·17	5·75	6·25	6·75
	pitch (feet)	4·25	3·5	3·5	3·33	3·5
	dev. area (square feet).	12·9	10·0	6·6	20·3	14·0
	number of blades	4	3	3	4	3
Revolutions		204	213	211	206	206
I.H.P.		134	150	131	175	178
Speed in knots		4·42	5·12	4·70	5·24	6·24

These vessels failed to get their speed of 6 knots on the first trials, and although alteration of propeller dimensions effected some improvement, the true solution was found in lowering the propeller by angling the shaft downwards until the blades projected below the keel, the results for which are given in column 6 of the table.

§ 81. Inward and Outward Turning Screws.—An inward turning screw is one in which the blades are moving inwards at the top of the screw disc when going ahead. Twin screws placed at some distance before the after-perpendicular must work in water which, near the hull at least, is not moving perpendicularly to the screw disc. This inward movement of the water is invariably greatest near the water line, *i.e.*, at the top part of the screw disc. The transverse movement in the stream flow is therefore in the same direction as that of inward turning blades at the top of the screw disc, when the screw is going ahead, and in the opposite direction to that of outward turning blades in the same position.

Taking first the case of screws working without any bossings or brackets in front of them. Froude's experiments show that, in so far as there is any difference, the inward turning has a slight advantage on hull efficiency, mainly due to a slight increase in wake factor. This advantage did not vary in any consistent way with spread of screws or type of vessel, and its mean value taken over a large number of cases is about 1·4 per cent., the mean increase in wake being ·6 per cent. Vessels having *A* brackets to support the outer ends of the shafts, with only small bossings or

"swellings" where these break through the hull, may be considered as working under conditions which approximate to the above, and the above conclusions will hold good in their case.

When large bossings are adopted the case is very different. These bossings are liable to interfere considerably with the natural flow of the water around the stern. This natural flow has been found to be upwards and inwards in more or less diagonal planes. A horizontal web at about half-draft prevents a good deal of this upward flow from taking place. The surface streams must then move in more rapidly than before, in order to avoid a cavity, and at the after end of the web where there is no restriction the lower streams will have a more marked upward movement. In the same way, a vertical web exaggerates the upward and forward flow along the buttock lines below the web, and there will be more rapid movement inwards just aft of the end of the web, particularly at the top of the screw disc.

The effect of such changes upon the towing resistance has already been given in Table 22. But this is not by any means the only effect of the bossings. The exaggerated inward flow near the surface with horizontal bossings and the equally important changes in stream flow due to vertical bossings, affect the wake and screw efficiency. The presence of the bossings in front of the screw will naturally increase the thrust deduction slightly, and, being a source of resistance, will increase the wake also, but it is in the effect upon the relative advantages of the two directions of rotation that it becomes important.

Fig. 48 shows the results of some experiments made in the Clydebank tank with bossings inclined at different angles on a model whose principal dimensions were :—

Length, 15·3 feet; beam, 2·5 feet; draft, ·75 feet; (M) value, 6·2.

Prismatic coefficient—fore-body, ·627; after-body, ·719.

Propeller dimensions—diameter, ·5 foot; spread, ·417 foot from centre line; screw centre before A. P., ·125 foot; immersion—tips, ·167 foot; centre ·417 foot.

Speed of model in knots

$$= \cdot 8 \sqrt{\text{length in feet.}}$$

The bossings were all ·125 × (screw diameter) in thickness, rounded at the outer edge, well tapered off aft.

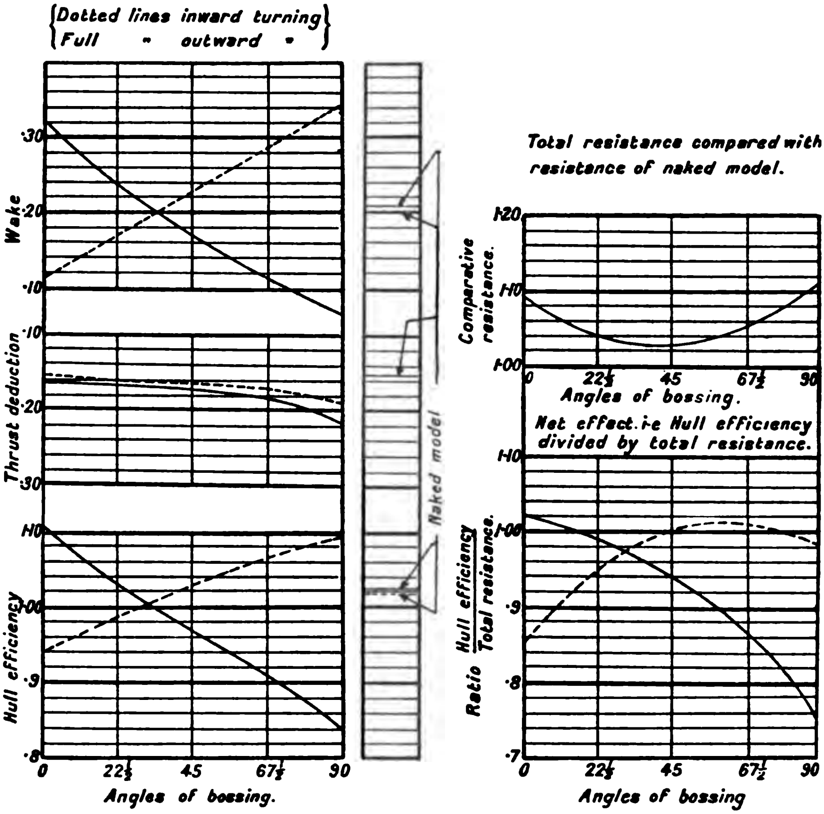


FIG. 48.—Angle of Shaft Bossing and Hull Efficiency Elements.

For all angles of the bossings the thrust deduction was increased some 2 per cent. over that for naked model, but the wake values differed with angle, the variation being in opposite directions with inward and outward turning. The exaggerated inward motion at the top of the screw disc produced by the horizontal webs gives an enlarged wake outward turning, and a decreased wake inward

turning. The vertical web has the opposite effect upon the wake. The effect upon the hull efficiency and the product of hull efficiency and total resistance, are shown by the figure.

These results are for model, and it is possible that the wake effects are somewhat greater than they would be in the ship, but there can be no doubt that the broad conclusions will be the same for both ship and model. Neglecting screw efficiency, the results warrant the conclusion that inward turning screws require an angle of bossing of about 45 degrees, and outward turning do best with smaller angles than this. Taking screw efficiency into account, as one must in a ship, it will be noted that with horizontal bossings and outward turning screws, or vertical bossings and inward turning screws, there is a high wake, which, with such forms as that under test, will mean either a very high slip and low efficiency of screw, or larger diameter with smaller pitch and larger bossings, the screw efficiency dropping, but not so much as before. This loss may be considerable, and Froude has stated that it just about balanced the gain in hull efficiency in cases he has tried (comparing the horizontal and 45 degree bossings). It follows that either inward turning combined with horizontal webs or outward turning and vertical webs, will always give bad results.

This to some extent explains the relatively bad results obtained by Mr. Duncan with inward turning screws and "fairly horizontal" bossings. With inward turning screws the speed obtained was 9·34 knots; by changing over the propellers so that they ran outward, and running the engines at the same revolutions as before, the speed was 10·34 knots. A similar case has been given by Mr. Taylor. The *Niagara II.*, a steam yacht of 2,000 tons displacement, 250 feet length, did 12·8 knots with inward turning screws, the I.H.P. being 2,100. With the same screws outward turning, the speed was 14·1 knots with 1,950 I.H.P. The vessel was fitted with "wide horizontal bossings."

Wake values for a great number of ships of various types are given in Tables 35 A, B, C, D.

§ 82. Multiple Screws.—The number of shafts adopted in propelling a vessel depends to a certain extent upon the type of

main engine to be used, and upon the actual power to be delivered by the screws. Large thrusts require large areas of screw blade, which means either large diameter with a given number of screws or smaller diameters and more shafts. But there are usually fairly rigid restrictions on the maximum diameter which can be worked on any ship. Long lengths of outboard shafting are very undesirable both from the resistance and vibration points of view, and generally the screw disc must come within the maximum beam and above the keel line. Screws of large diameter require high shaft lines in the hull and therefore reduce the cargo space a little, and in certain vessels would require more care in laying the vessel alongside a jetty to avoid fouling the screws. On the other hand, the fewer the number of shafts the less expensive, as a rule, is the installation, the supervision is less, and the shaft bossing resistance is less.

Assuming that the main engines require high revolutions for good efficiency, it is known that the possibility of getting good efficiency out of the screw is limited by two things :—

The thrust cannot be increased beyond a certain amount per square foot of blade surface.

The pitch must be such as to give a reasonable slip.

But it is seldom good to keep the pitch very low relative to diameter, and as the power to be delivered by the screws is increased, there will come a time when it is advisable to increase the number of shafts rather than to increase the diameter or pitch of the existing propellers.

With low revolutions of the main engines a large diameter of propeller and high pitch ratio can be adopted, and higher thrusts can be delivered on a single shaft with quite reasonable screw efficiency. It is of little or no use adopting a type of main engine suitable for high revolutions, and sacrificing considerable efficiency by working it at low revolutions to suit the screw. The gain, if it exists, would not warrant the extra cost usually involved. If an economical high revolution engine is to be used, its revolutions must be reduced to what gives reasonable efficiency of the screw, by some means external to the engine.

The propulsive efficiency of triple or quadruple screws can be obtained exactly the same as for twin screws. It is necessary to know the wake factor and hull efficiency for each screw position. With these the screw efficiency can be obtained from Froude's results (Figs. 37 and 40). The doubtful point at present is the hull efficiency. The best data upon this for quadruple screws is given in a recent paper by Mr. Luke. A comparatively fine lined model was tried in the Clydebank tank with twin and quadruple screws. The particulars of the model and screws are given in the table.

TABLE 28.

Quadruple Screw Experiments.

Model dimensions :—		
Length	16·67 feet	
Beam	2·5 ,,	
Draft	·75 foot	
Block coefficient	·60	
Propeller particulars :—	Inner.	Outer.
Diameter (feet)	·333	·333
Number of blades	3	3
Spread from centre line (feet)	·25	·583
Centre before A. P. (feet)	·25	1·5
Clearance from hull	·125 diam.	·125 diam.

The inner and outer screw discs when projected on the transverse plane just touched each other.

Experiments were made with the forward and after screws separately, each giving a thrust equal to half the resistance of the model plus the augment caused by these screws. The aggregate thrusts of the forward and after screws thus equal the total augmented resistance. This assumes complete non-interference, and as this was a doubtful point, further experiments were made to observe the effect on the performance of the forward screws, of a pair of revolving screws placed in the after position, and also on the after screws of a pair placed in the forward position.

The results of the twin-screw experiments call for no special comment except that the high hull efficiencies are probably due to the close proximity of relatively small propellers to the hull of a very wide model. Table 29 gives the hull efficiency elements

for each position with the four screws revolving. It will be observed that the forward screws have a material influence on the wake of those in the after position, and the hull efficiency of these screws is considerably reduced; but the after screws have very little effect upon the forward ones. Some light upon two most important points in connection with the propulsion of vessels by quadruple screws is obtained from these experiments.

TABLE 29.

Results of Quadruple Screw Experiments.
(Model and screw data are given in Table 28.)

—		Wake.	Thrust Deduction.	Hull Efficiency.
After screws only	outward turning .	·21	·14	1·04
	inward turning .	·20	·11	1·07
Forward screws only	outward turning .	·24	·18	1·08
	inward turning .	·24	·10	1·12

The effect of action of neighbouring screws on above results:—

—			Wake.	Thrust Deduction.	Hull Efficiency.
Wake, etc., of after screws	outward turning	forward screws (outward turning) .	·16	·18	1·01
		forward screws (inward turning) .	·18	·12	·99
	inward turning	forward screws (outward turning) .	·15	·12	1·01
		forward screws (inward turning) .	·10	·10	·99
Wake, etc., of forward screws	outward turning	after screws (outward turning) .	·22	·18	1·06
		after screws (inward turning . .	·22	·12	1·07
	inward turning	after screws (outward turning) .	·28	·12	1·08
		after screws (inward turning) . .	·28	·10	1·10

In the first place the relative wake values to use in propeller design are approximated to, and in the second place the combinations of directions of rotation to ensure the highest gross hull efficiency can be obtained. With this particular model, the gross hull efficiency values obtained with the various possible combinations of inward and outward turning screws in either position are almost identical, although it is not possible to say that this is a general rule. The fact that the clearances and diameters were the same for both positions, and that the effect of the bossings is to be superposed upon these results, must be borne in mind when using them in any case.

In several ships whose trials have been analysed the author has found that the forward screws have influenced the wake of the after ones, reducing it to about equality with that of the forward, and in the case of a high-speed triple-screw vessel to 4 per cent. less than that of the wing screws, the wake of which was 2 per cent. positive.

This influence of the forward screws upon the after ones cannot be good for the screw efficiency itself. It would only be felt upon the outer part of the screw disc, and the blade tips would pass through this sternward wash from the forward screws, into the strong forward wake near the ship's hull, with every revolution. When projected upon the transverse plane, in the case of triple screws, the discs should clear each other by at least ·2 of the diameter. With quadruple screws good results have been obtained when the projections of the screws on a transverse plane only clear each other by a few inches, but as a rule the forward screws have been several diameters ahead of the after ones and somewhat higher in the hull. The natural stream line flow will tend to wash the race from the forward screws upwards as well as inwards, and the raised position of the forward screws combined with the distance between the two positions, all tend to help the race of the one to clear the other.

The results of some model experiments made in the Spezia tank are given in Table 30. These experiments were made in order to compare triple and quadruple screws. It will be seen that the

mean wake values are higher for three screws than for four, and this was largely due to an increase in the wake of the centre screw of about 7·8 per cent. As the thrust deduction was about the same with either three or four screws, the hull efficiency was therefore better with the former. Triple screws also had the advantage of lower appendage resistance, and would have a decided advantage in propulsive efficiency in this case, if they could be designed to avoid cavitation and do their work with as high screw efficiency proper as quadruple screws would give. This would need investigation for any ship on the lines already given, before a complete comparison could be made.

TABLE 30.

Form :—	A				B			
Length in feet	630				620			
Beam in feet	93·3				90·0			
Draft in feet	27·6				27·9			
Displacement in tons	24,700				24,600			
Block coefficient	·536				·555			
Number of screws :—	4		3		4		3	
Transverse spread of screws.	Projection of discs nearly touching each other, centre of outer screws 3·7 diameter apart.		Projection of discs just touching each other.		Centres of outer screws 4·2 diameter apart, inner screws 1·6 diameter.		Wing screws 2·64 diameter apart.	
Centre of screw from after end of water line (cruiser stern) — outer	111 feet		77·4 feet		86·3 feet		62 feet	
— inner	61 „		42·6 „		44·0 „		37·5 „	
Ratio of resistance of naked ship to resistance with appendages, corrected for ship	·925		·95		·95		·975	
Speed in knots :—	12	24	12	24	12	24	12	24
Mean wake values	·12	·09	·17	·13	·10	·10	·18	·18
Mean hull efficiency	1·05	·96	1·10	·98	·98	·95	1·09	1·0
Product of hull efficiency and appendage coefficient	·971	·888	1·004	·93	·93	·903	1·06	·975

CHAPTER XXII

MAIN ENGINE

§ 83.—It is only intended here to run over the main effect which type of engine has upon the propulsive efficiency in general, and not to consider the reasons for this effect so far as the engines are concerned. Every type of engine, whether it is steam reciprocating with three or four cranks, steam turbine, oil, gas, or electric, has a certain range of revolutions over which it will work at its best economy, and there is a penalty to pay, if for any reason the revolutions are fixed outside this range. In some types this penalty is a comparatively small one, and in others assumes serious proportions.

There are two different sides to the question which have to be considered :—

(1) The cost of producing one brake horse-power per hour on the propeller shaft ;
(2) The economical utilisation of the whole power developed by the engines.

Good propulsive efficiency is of little practical use if it is obtained, either by the wasteful consumption of cheap fuel and water, or by the economical consumption of an expensive fuel. On the other hand, a cheap means of producing power has to be combined with an efficient mechanism for thrusting the ship along, or it is of no use to the naval architect.

For very large powers only two types of engine are available, namely, the reciprocating and turbine engines, and of these the reciprocating is approaching its upper limit both as regards piston

speed and revolution. For more moderate powers the oil engine becomes a competitor, and for still smaller powers the gas and electric engine are available. The power limitations of the last three types are not necessarily set by any inherent qualities of the engines, but because of their comparatively smaller development.

The Reciprocating Engine is fairly efficient at all moderate speeds, its mechanical efficiency varying from ·8 for low to ·88 for full power in triple-expansion, and to ·92 for quadruple-expansion engines, exclusive of thrust block friction, which might be taken as absorbing 2 per cent. of the power. Its coal consumption is about 1·3 lbs. per I.H.P. per hour for a three-crank engine, exclusive of auxiliaries, or 1·54 lbs. per S.H.P. Taken on a prolonged voyage and for all purposes, the consumption of coal may be taken as roughly 1·5 lbs. per I.H.P. for triple and 1·34 lbs. for quadruple-expansion engines. This gain in coal consumption with the quadruple-expansion engine has to be balanced against the extra cost and larger engine-room involved, and would be realised only on long voyages.

Going astern its power is about ·89 of that going ahead.

The Turbine.—For high powers to be delivered at high speeds, the turbine has a lower coal consumption per S.H.P., and its efficiency is at least equal to, and sometimes more than that of the quadruple-expansion engine. It produces an almost uniform torque on the propeller shaft, enabling higher mean thrusts to be used on the propeller blades and causing less vibration. On the other hand, it is not reversible, and an astern turbine or a transformer must be fitted. A turbine gives better efficiency as the blade speed approximates to the steam speed, and for this reason high revolutions are better than low. A land steam turbine developing powers of 10,000 H.P., gives the best performance at 1,000 revolutions per minute or above, and for smaller powers at still higher revolutions, the coal consumption being as low as 1·0 lb. per S.H.P. per hour.

But if a vessel is of only moderate speed the turbines cannot be run at such high revolutions without serious loss in screw efficiency, and to avoid this loss transmission gear between turbines and screw has been adopted in recent years.

This loss at low speed can be seen from the following results for the *Amethyst* and *Topaze* :—

TABLE 31.

Distance Travelled per Ton of Coal.

Speed in Knots.	*Amethyst*, with Turbine Engines and direct Drive to Screws.	*Topaze*, with Four-crank Reciprocating Engines.
10	7·42	9·75
14	6·6	6·8
18	4·8	8·7
20	4·2	2·9

Trials with other vessels show generally the same variation. Thus at one-fifth power the "Invincibles," with turbines and direct drive to screws, require 2·4 lbs. of coal per S.H.P., and the "Minotaurs," with reciprocating engines, require only 1·9 lbs. At high speeds the corresponding figures are 1·2* lbs. and 1·75 lbs. respectively. The water consumption varies in the same way, but not to the same extent, the figures at high speed being about 12·0 lbs. and 15·8 lbs. respectively.

All of this gain in the turbine at high speed shown by the consumption per S.H.P. is not necessarily realised. Larger powers are required owing to the lower propulsive efficiency compared with that obtained with reciprocating engines at lower revolutions. The propulsive efficiency of the *Utah*, with turbines (taken as the ratio $\frac{\text{E.H.P.}}{\text{S.H.P.}}$), is ·62, and that of the *Delaware*, with reciprocating

* The coal consumption of the *Lusitania*, with Scotch boilers and turbine engines, is 1·43 lbs. per S.H.P. per hour.

engines, is ·69, assuming that $\frac{\text{S.H.P.}}{\text{I.H.P.}}$ is ·90. This loss* in propulsive efficiency must be set against any gain in consumption per S.H.P. In some cases it exceeds the latter. A comparison of the results of the *Caronia* and *Carmania* will illustrate this. The former has quadruple-expansion engines driving two screws at 80 revolutions per minute at 18 knots; the latter has compound turbine engines driving three screws at 175 revolutions per minute. The coal consumption of the latter is stated to be "considerably greater" than that of the former.

The crossing-over point at which the purely reciprocating and purely turbine ship are about equally economical in coal appears to be in the neighbourhood of 15 to 18 knots for the intermediate class of passenger-cargo steamers. For steamers of moderate power and speed, the two propulsive defects of a turbine with direct drive to the screw—non-reversibility and high revolutions—can be removed *wholly* by fitting a transformer of the "Fottinger" or electric type between the turbine and propeller, or in part by the introduction of mechanical gearing. The relative propulsive advantage of the reciprocating and turbine engine, with or without transmission gear, depends to some extent upon whether the former can be arranged to give the best screw efficiency. If this is the case, whatever is to be gained by change of type of engine must be obtained from the turbines and transmission, and to whatever extent it is not the case there is further possible gain with the latter. The additional weight of the transformer, whether water pump, electrical, or mechanical gearing, is roughly speaking compensated for by reduction of weight of turbines. In some cases there has been a fair gain in weight by fitting a high revolution turbine and a transformer of some kind,

* The Italian cruiser *San Marco*, with turbines on four shafts, has practically the same propulsive efficiency as the sister ship *San Giorgio*, with reciprocating engines on two shafts, the S.H.P. of the former being ·89 of the I.H.P. of the latter at 22 knots, the mean revolutions per minute being 380 and 140 respectively.

but the area of engine and boiler room remains about the same in vessels of moderate speed.

The "Fottinger" Reducing and Reversing Gear can be arranged for reduction ratios from 3 to 1 up to 12 to 1 and has the following efficiency :—

Small powers about 150 H.P. . .	efficiency = ·865
Larger powers about 600 H.P. . .	„ = ·88
Powers above 1,000 H.P. . . .	„ = ·90

The above efficiencies *include* the effect of thrust block friction. A small portion (1½ to 2 per cent.) of the loss in the transformer can be recovered by using transformer water for feeding the boiler. No astern turbine is required in this case, the transformer taking its place, and in a recent case the astern power has been as much as 80 per cent. of the ahead power. This astern power can be maintained for prolonged periods without detriment to the engines.

With an installation developing 2,400 S.H.P. the water consumption was 12·46 lbs. per S.H.P. for turbines and transformer alone, and the coal consumption was 1·38 lbs. per S.H.P. including auxiliaries, the coal not being of very good quality. If the transformer is to be any advantage in propulsion, the known loss in it of about 11 per cent. must be made good by greater efficiency in the screw and turbine. The above figures, which are believed to be reliable, compare very favourably indeed with the water and coal consumption of a reciprocating engine, and the efficiency of the screw with the turbines and transformer would be at least as good as with the reciprocating engines, and much better than with direct turbine drive.

Installations of 10,000 H.P. on one shaft are in hand for the 25,000 ton steamer *Admiral von Tirpitz*, and the shop tests show an over-all efficiency of 92 per cent. for these.

Mechanical Gearing.—Gearing up to S.H.P. of 2,000 on each pinion and to 4,000 per shaft by working two pinions to one gear wheel is now in use on the *Anyo Maru*, and pinions to take up to 3,000 H.P. are stated to be under consideration. Reduction

ratios varying from 20·0 to 4·5 are in use. The loss of power in the gearing is about 2 per cent. and there is an additional small loss in the ordinary thrust block, which may bring the total loss to 4 per cent. or 5 per cent. If pivoted blocks are used in the thrust bearing, the loss in it is very small, being from ·2 to ·3 per cent. The gain in efficiency of the turbine by running it at the high revolutions possible with the gearing is considerably greater than the above loss, and the screw can be arranged to work at its best efficiency.

This eliminates the most serious disadvantage of the turbine for low and moderate speed vessels, but the astern turbine is still a necessity and leads to some complication, and as a rule to loss of astern power, compared with the reciprocating engine. The astern power is usually about 60 per cent. of the ahead power, but more can be arranged if desired. The cost of the installation is from 5 per cent. to 10 per cent. greater than that of reciprocating engines of the same power and somewhat larger boilers.

The gain in consumption can be seen from the results of the trials of the *Cairngowan* and *Cairnross*. These are sister ships of length 370 feet, beam 51 feet, block coefficient ·784 and ·779 respectively, having a sea speed of 10 knots, the boilers and propeller being the same on the two ships. The former had a triple-expansion engine, the latter high-speed turbines with Parson's mechanical gearing between the turbine and propeller shafts. The coal consumption of the former with all auxiliaries was 15 per cent. higher than that of the latter with the propellers making the same revolutions. The S.H.P. of the latter was 87·7 per cent. of the I.H.P. of the former, and, using this figure in the *Cairnross* to calculate the equivalent I.H.P. for a reciprocating engine, the lbs. of coal per I.H.P. for all purposes becomes 1·45 and 1·70 for the geared turbines and reciprocating engines respectively. The *Cairnross* had somewhat finer lines, which no doubt helped her, but, making all due allowance for this and for the comparatively high consumption of the reciprocating engine, there is still a balance of about 9 per cent. in favour of the geared turbine.

Electrical Transformers.—These have been tested on several ships of comparatively moderate powers. The efficiency of the electrical gear is claimed to be 88 per cent. Just as with any other transformer, the turbine can be run at very high revolutions. With electrical gear, however, the ship's speed can be altered or reversed with the turbines running in their normal direction. No astern turbine is required with it, and the full power of the main engine is available for going astern. For vessels of high power a number of independent high-revolution turbine generators can be used in any convenient part of the ship, and some of these can be cut out for low speeds. The system can be used with internal combustion or any other engine as the prime mover. It has the disadvantage of introducing high-voltage alternating current into the engine room. The United States coal collier *Jupiter* (length, 520 feet; displacement, 19,000 tons loaded) is fitted with one turbo-generator making 1,990 revolutions per minute at full speed (14 knots estimated), driving two motors with an approximate reduction ratio of 18·0. The shop tests of turbine and generator show a steam consumption of 12 lbs. per S.H.P. The weight of machinery is said to be 156 tons compared with 280 tons in the sister ship *Cyclops*, driven by two triple-expansion engines, and she has maintained 14·8 knots on her trials against 14·6 knots of the latter, the S.H.P. of the former being 6,300 and the I.H.P. of the latter 6,700.

The cargo boat *Tynemount* is fitted with two 300 B.H.P. oil engines making 400 revolutions per minute, supplying current to one 500 B.H.P. induction motor on the propeller shaft. This vessel is said to have passed her trials satisfactorily. Consumption and efficiency figures have not been published.

Combination of Turbine and Reciprocating Engines.—In this system the steam is first taken to a reciprocating engine and then to a low-pressure turbine. Each type of engine is then run under its best conditions, and large expansions of the steam are possible. No astern turbine is required with this arrangement, but the astern power is necessarily limited to 89 per cent. of the ahead power of the reciprocating engines. Unless there is sufficient

beam to place them more or less abreast each other, the engines are liable to require a long engine-room, and cargo and passenger space then suffers.

The system is most useful for vessels of moderate speed (15 to 21 knots) and fairly large power. Comparatively high revolutions are necessary for economy, the drive being direct from turbine to screw as a rule. The saving in coal in a good example is given by the comparison of the *Laurentic* and *Megantic*. These vessels are 565 feet in length, 20,000 tons displacement, the former having triple-expansion engines on the wing shafts and a low-pressure turbine on the centre shaft. The latter has quadruple-expansion engines with two screws, and consumes 12 per cent. more coal than the former. The same system is fitted in the *Otaki* (length, 465 feet, displacement, 11,750 tons on trial), and the advantage in its favour compared with quadruple-expansion engines is stated to be 20 per cent. in steam consumption. The *Olympic* (length, 852 feet, speed, 21 knots) and several other large vessels making long voyages also have such engines.

Oil Engines.—The oil engine has the great advantage that it is self-contained, requires no intermediate process of transformation between fuel and engine, has low stand-by losses, and requires smaller space than the reciprocating engine and boilers. The fuel can be quickly stored in places not suited for coal and requires no trimming. Against these advantages must be set the facts that the supply is only possible at certain ports, and the cost per ton is much higher than that of coal, outside of certain restricted areas. Low flash-point oil cannot be carried under holds owing to the danger of vapour getting into the latter.

The power which can be generated in one cylinder is limited at present to about 500 B.H.P., and by grouping smaller cylinders powers up to 2,000 B.H.P. can be developed on one shaft. The type involves reciprocating parts, with the possible attendant vibration. The two-stroke is more simple and has a more equal turning moment than the four-stroke, but its consumption is about 10 per cent. higher and the piston temperatures are higher. Engines of moderate power—up to 1,250 S.H.P. on each shaft—

have now shown themselves to be fairly reliable on service. The oil consumption and efficiency can be seen from the following figures:—

The *Selandia* (length, 370 feet, displacement, 9,800 tons loaded) is fitted with two shafts each driven by a four-stroke cycle "Diesel" oil engine making 140 revolutions per minute at about 11 knots, developing 2,500 I.H.P. The mechanical efficiency of one of the main engines was found to be ·85. This is exclusive of thrust block friction and auxiliaries, and in comparing it with the similar figures for the steam reciprocating engine it should be reduced some 3 to 5 per cent. to allow for the latter. The oil fuel consumption including auxiliaries was ·44 lb. per S.H.P.

The *France*, (length 430 feet, displacement, 10,700 tons loaded) is fitted with a two-cycle engine on a centre line shaft, and makes 230 revolutions per minute, developing 1,450 B.H.P. at a speed of 10·2 knots. The mechanical efficiency of the engine was found to be ·71, the engine being run on lamp oil and consuming ·47 lb. per S.H.P.

In considering the application of oil engines to low-speed ships the effect of revolutions upon efficiency of the propeller must be carefully borne in mind. Increase of revolutions above a certain limit, which depends upon the power and speed, is bound to lower the screw efficiency. Developing the power on two shafts, as in the case of the *Selandia*, raises this revolution limit considerably above its value for a single shaft transmitting the whole power at the same speed.

Gas Engines.—No engine of high power for use on board ship has yet been developed. Such an engine for general use requires a producer to burn bituminous coal on a large scale without caking, and the absence of such an one of reasonable proportions and weight stands in the way of any considerable advance. The majority of gas engines are dependent on a clutch for reversing, and special means are required for setting the engines in motion. The stand-by losses are about the same as for a vessel with an ordinary steam boiler.

Where a good supply of coke, coke breeze or anthracite coal

is available this type of engine is used with great fuel economy. For small powers — up to about 450 H.P. — the consumption is ·8 lb. of anthracite and 1 lb. of bituminous coal per I.H.P. per hour.

Effect of Revolutions.—In the preceding section it has been stated that the propulsive advantages which might be gained by the adoption of certain types of prime movers are affected (sometimes adversely) by the revolutions which are necessary to obtain reasonable efficiency in the engine. Two cases have been chosen to illustrate this effect. One of these represents a comparatively low-speed vessel, the other a vessel of intermediate type, both having twin screws.

SHIP PARTICULARS.

Item.	Case 1.	Case 2.
Speed in knots (V)	15·0	20·6
Thrust horse-power to be developed by two screws ($2H$)	7,500	16,000
Wake factor	·18	·144
Speed in knots through wake water (V_1)	12·71	18·0
Minimum area in square feet for cavitation (assuming 12 lbs. to the square inch)	56	84
Screws three-bladed of uniform pitch.		

For both cases, diameters and efficiencies for several pitch ratios have been calculated for various revolutions, and are plotted in Fig. 49. For any revolutions of the engine, the diameter and efficiency of the propeller will be given by the curves for the particular pitch ratio chosen.

When considering the results two things have to be borne in mind—first, that the efficiency curves are for screw only, and any loss in this direction must be balanced against the lighter and smaller engine possible at the higher revolutions, and a better efficiency of engine in the case of turbine drive; secondly, that the small diameters would be associated with smaller bossings or brackets, which gives them a slight advantage, and with proper

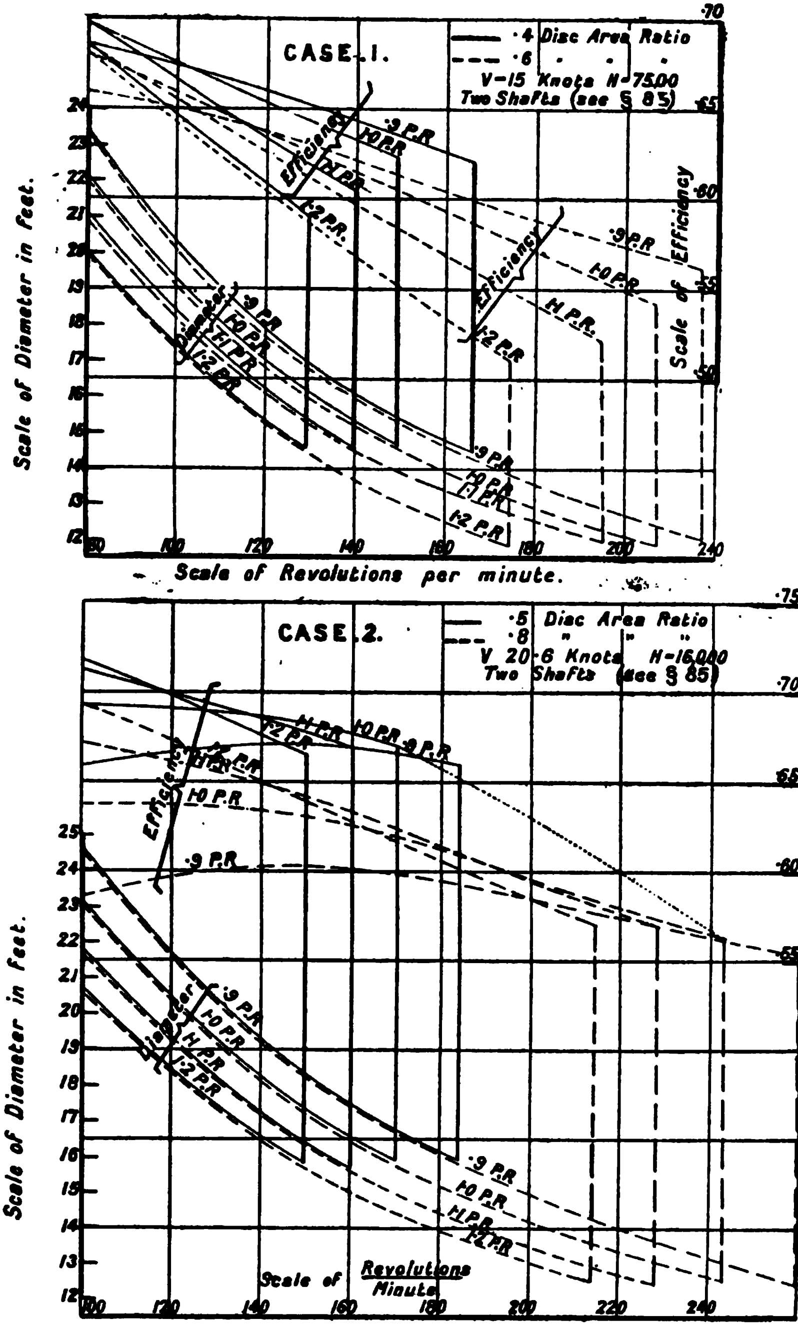


Fig. 49.—Revolutions and Screw Efficiency.

Note.—The figures on the curves are effective pitch ratios, equal to 1·08 times the face pitch ratio.

clearances there is no reason to suppose there will be any loss in hull efficiency with them. Somewhat more clearance will be required with the small screws than with the large ones. There is no definite published data on this point, but as both screws will be developing the same total thrust, the *intensity* of the water pressures around the smaller screw will be greater, and more clearance will be required to avoid an increase in the thrust deduction factor, which would counterbalance the gain due to the higher wake factor with the closer set screws.

Case 1.—The developed blade area per screw must exceed 56 square feet, corresponding to diameters of 14·6 feet and 12·0 feet for ·4 and ·6 disc area ratios respectively. These limits are shown by the thick vertical lines. As revolutions increase, the maximum possible efficiency steadily decreases, until the cavitation limit is reached. It is then necessary to increase the disc area ratio, if further increase is made in the revolutions, and this entails a more rapid drop in the efficiency. The curves also serve to show that moderate changes in the pitch ratio of a screw of fixed diameter and area have little effect on efficiency down to diameters of 17 feet. For smaller diameters, pitch ratio must be decreased as revolutions are increased, in order to obtain the best efficiency.

Case 2.—The minimum blade area for avoidance of cavitation in this case, gives diameters of 15·9 feet and 12·5 feet for disc area ratios of ·5 and ·8 respectively, as shown by the full vertical lines. As before, the maximum possible efficiency falls off as revolutions are increased, and increase in the disc area ratio carries with it a still more noticeable drop in efficiency. This is shown for the 1·0 pitch ratio by the fine dotted line, which holds for screws of fixed blade area with decreasing diameter as revolutions are increased. It will be seen that very small pitch ratio is never good in this case, and is particularly bad when associated with low revolutions or large area ratios.

In both cases, variation of blade area has little effect upon the diameter or revolutions for given pitch ratio.

These curves are based upon the data given in Figs. 37 and 39;

they have been worked out as shown by the example in Table 25, and other cases can be dealt with in the same way. Such separate treatment is necessary in any case where there is any departure from general practice, particularly if there is any liability to cavitation.

CHAPTER XXIII

CAVITATION

§ 84.—"Cavitation" is the name given to the phenomenon which makes itself felt by the absence of proper increase in screw thrust with increase of torque. The cause of it is not thoroughly understood, but the following explanation appears to be sound, and does not run counter to good theory :—

Reynold's experiments show that the admission of air to a screw causes it to race heavily. If this air is carried round by the blades, it is compressed, and when released from the pressure expands and pushes the water back and out as well as forward, and thus retards the flow through the screw. If, therefore, the blade tips cut the water surface, or are near enough to the surface to suck down air, loss of efficiency will result. Under these conditions the thrust and revolutions of the screw are very sensitive to its immersion, and slight variations of the latter, caused by pitching or the passage of waves, will tend to cause considerable variation in the revolutions, producing what is commonly called "racing of the engines." This "racing" is a species of cavitation, but all cavitation is not due to the blades breaking the surface or sucking down air. Screws have been known to cavitate badly when there has been no question of the water surface being broken. Such phenomenon may be due to several causes.

Cause 1.—An advancing and rotating screw produces in front of it a suction which causes the water to move towards the screw. The movement of the water is mainly longitudinal, and the acceleration of the particles at any point depends upon the difference of the still-water pressure and the suction of the screw at that point. This still-water pressure is that due to the depth of water, added to the atmospheric pressure, which averages

14·6 lbs. per square inch. Thus at 10 feet immersion the pressure is 19·4 lbs.

When the suction *at any point* has reached the still-water pressure, increase of rotation will not produce more suction, and it follows that the acceleration of the water particles will not be increased, or, in other words, the water supply to the screw at this point remains the same although the revolutions have been increased. One of two things may then take place, either the water will rotate partially with the screw blade or a cavity will tend to form. The formation of the cavity will cause a break up of the stream flow at the point and consequent eddy-making. Both the rotary action and the formation of cavities mean loss of efficiency.

If every part of the screw blade surface exerted precisely the same amount of force, this limiting suction would be attained all over it. But the water which passes the screw blades is forced into more or less definite stream lines, and the pressures in these streams vary according to their distance from the blade surfaces and screw centre. The thrust which the screw exerts is made up of two parts—viz., the suction on the back and thrust on the face. Owing to the circumferential velocity these are greatest near * the tip, and, owing to the stream line action, at any radius the pressure across the blade is greatest near the leading edge. The greatest suction will therefore occur at the back of the blade towards the tip of the leading edge, and the greatest thrust at the same position, but on the face of the blade. *If the back of the blade is very full near the leading edge*, it tends to produce at all small slips a negative pressure or suction on the *driving* face near the leading edge, and such a feature should be avoided.

For most blades tested lineally in air at small angles, this suction effect constitutes more than half the thrust of the blades, and there can be little doubt that the efficient working of the

* Some recent experiments with blades in air show that the maximum forces are not experienced at the tip of the blade, but at some distance in. For a rotating blade this distance would be approximately one-eighth to one-sixth the radius of blade from the tip.

screw depends upon the attainment of good suction on the back of the blades. Since cavitation will take place when the suction *at any point* reaches a certain limit, the back of the blade must be such as to attain good suction without any marked high *local* value. For this a sharp leading edge and a rounded back with no sudden change of shape are required. The driving face appears to make little difference to the phenomenon provided it is either flat or hollow in section, *i.e.*, of uniform or gaining pitch passing from leading to following edge. Since all corners and abrupt features tend to produce local high velocities, it follows that the contour of the leading edge at the blade tip must be well rounded to avoid cavitation. Above a certain limit (when the thickness exceeds ·1 of the width of the blade) the thicker the blade the smaller is the slip angle at which these high local suction values are likely to be attained, and the earlier is the break-down in the efficiency of the screw.

Cause 2.—The above assumes that the screw is rotating uniformly, so that the *maximum* cavitation pressure limit can be reached by the screws. This is fairly representative of turbine screws of small diameter, whose revolutions are generally constant. But with reciprocating engines the rate of rotation of the screw varies considerably during a single revolution. This can be seen from Table 32, p. 219, giving the results of measurements taken by Mr. G. H. Heck.

This variation in rate of rotation affects the slip at which the screw works, and hence its thrust. Thus with a mean slip of say 20 per cent. a 2 per cent. increase in revolutions brings the slip to 21·6 per cent. and increases the thrust of the propeller 12 per cent. The mean thrust of the propeller is therefore less than its maximum by an amount depending upon the variation in rate of rotation during a revolution. To a similar extent, the thrust when cavitation becomes present is less than what it might be with uniform angular velocity.

The following table shows that the main factors in obtaining uniform rotation, are good balance of engines and good immersion. High revolutions also help in this direction, due to the

inertia of the rotating mass, particularly in the case of turbine engines.

Table 32.

Variation in Rate of Rotation of Screw Shafts.

Type of Vessel.	Type of Engine.	Revolutions per Minute.	Propeller Tips.	Percentage Variation from Mean of Revolution.	
				Taken over $\frac{1}{8}$ Rev.	Taken over $\frac{1}{4}$ Rev.
Cargo-boat .	two-crank compound.	58	well immersed.	12	8·6
Cargo-passenger.	triple-expansion three-crank.	66	well immersed.	4·6	2·1
"	"	68	24 inches out of water.	7 to 9 (not racing)	
Large cargo .	three-crank .	71	20 inches out of water.	5·6	2·15
Large cargo .	four - crank balanced.	66	55 inches out of water.	4·6	1·2
Small high-speed passenger.	four - crank balanced.	104	immersed 4 inches.	4·8	1·7

Cause 3.—A good clearance between hull and blade tip is a necessity for any screw working at moderate thrusts. With reduction of clearance the supply of water to the blades as they pass the hull becomes more restricted, and eventually becomes insufficient. Each blade then breaks up the streams in the effort to supply itself with water. This water is then not thrust backwards, but whirled round by the blade, which for some portion of its revolution, after passing the hull, has every appearance of "cavitating." It is nearly always possible to tell if this is taking place, if one can get to the inside of the hull plating near where the screw blades pass. The plating is subject to sharp and rapid vibration, and an emphatic crackling noise can be heard as the water is broken up.

The higher the tip velocity of a screw the greater become the forces involved and the larger the clearance it requires. For this reason a section of the hull which follows the sweep of the pro-

peller tip for any distance, is likely to cause trouble unless the clearance is considerable. In destroyers with clearances of only 10 inches, this phenomena has been obviously present, and it has been observed in larger ships having the same and slightly greater clearances. Turbine-driven screws having 30 inches clearance have not shown this defect although cavitating for other reasons.

Cavitation Pressure Limits.—With high revolution, directly driven turbine screws, well immersed, having a clearance of at least 24 inches and good easy buttocks to the hull, a thrust of 13·2 lbs. per square inch can be allowed. With the same conditions but deeper immersions 13·5 lbs. may be taken, and reputed pressures of 13·8 lbs. are said to have been realised. With a clearance of 12 inches a pressure of 13 lbs. can be realised under favourable conditions (the screw tips being immersed at least 2 or 3 feet).

With a well-balanced quadruple reciprocating engine running at high revolutions, Barnaby found that the pressure limit was given by :—

$$\text{Pressure per square inch of projected area} = 10{\cdot}85 + \tfrac{3}{8} \times h \text{ lbs.}$$

where h is the immersion of blade tip in feet. These figures are considerably lower than the preceding, but are based upon trials with the *Daring*, in which the blade tips were immersed only 1 foot and the hull clearance was about 10 inches.

For propellers driven by four-cycle explosion engines, in which the turning moment varies considerably, Barnaby gives a figure of 8 to 9 lbs. per square inch, unless a large number of cylinders is used.

CHAPTER XXIV

MEASURED MILE TRIALS

§ 85.—Trials upon the measured mile are intentionally made under the best possible circumstances as regards weather and sea and with a clean bottom. Under such equable conditions the performances of different vessels may be compared with one another and with tank results.

Most large vessels are now tested by a series of "progressive trials," *i.e.*, trials made at a series of speeds from the lowest to the maximum possible. A single trial at high speed serves but very little use, except to show that a certain engine power and general efficiency (as may be laid down in terms of power and speed in the ship's contract) have been obtained. Progressive trials, if properly analysed, can be made to give valuable data, not only as regards the ship tested, but such as will be useful in future designs. The real object of such trials is to measure the propulsive efficiency of the vessel, and for this complete records of revolutions of each shaft, ship speed, and indicated or shaft horse-power are required. All the trials should be made at as near as possible to the same displacement, which should be carefully checked by taking the drafts before leaving and on returning to the moorings. In the case of high-powered vessels, if the trials at different speeds are run on the same day after each other, the time between each set should be noted, in order that the displacement may be corrected for the fuel and water consumed during the intervals.

Every care is required in carrying out such trials to see that errors shall not creep into the results. With good judgment

many of these can be avoided. The more important of them are :

(*a*) Insufficient length of run, after making the turn to go over the course again. The vessel must attain uniform speed and be on its course parallel to the posts before starting on the mile, otherwise the speed will be low.

(*b*) Variation in tide. The tide on the mile is never constant either in direction or in speed. It can be eliminated from the measured speed by making an even number of runs on the mile, and using the mean-of-means method of obtaining the average speed. If an odd number of runs is used to deduce the speed the result will be in error, being on the small side if the larger number of runs is made against the tide.

(*c*) When there is any variation of tide at different parts of the measured course, the runs both with and against the tide should be made in the same part of the channel.

(*d*) The steam valves should not be touched once the vessel has settled on her course and is approaching the mile.

(*e*) The rudder should not be put over more than 2 or 3 degrees whilst the ship is approaching or on the mile.

Analysis of Data.—The records of speed, revolutions, and power obtained during the trials should *all* be averaged by the mean-of-means method. Thus, if four runs are made at approximately the same speed the average is obtained as follows :—

Measured Speed or Power, etc.	First Mean.	Second Mean.	Third Mean and Average.
x_1	$\frac{x_1+x_2}{2}$		
x_2	$\frac{x_2+x_3}{2}$	$\frac{x_1+2x_2+x_3}{4}$	$\frac{x_1+3x_2+3x_3+x_4}{8}$
x_3	$\frac{x_3+x_4}{2}$	$\frac{x_2+2x_3+x_4}{4}$	
x_4			

The arithmetic mean, viz., $\frac{x_1+x_2+x_3+x_4}{4}$, differs from this a little and is not so correct as the above. It is important that the speeds used in the calculation are consecutive and not selected from a number of results.

The trial results having been averaged can be plotted in the following form (for each shaft separately when there is any marked divergence between them) :—

(1) A curve of $\dfrac{\text{I.H.P. or S.H.P.}}{(\text{displacement})^{\frac{2}{3}}V^3}$.

(2) A curve of $\dfrac{\text{revolutions per minute} = R}{\text{ship speed in knots} = V}$.

Knowing the pitch of the propeller (this being as set in the shop or as measured in place on the shaft when in dry dock), the latter curve can be put in the form

$$\frac{R \times P}{V}.$$

With a good form of vessel neither of these curves should vary in ordinate value very much. If the former shows a tendency to increase slowly with speed (as it does for full low-speed vessels), the latter should also show a tendency in the same direction. The $\frac{\text{I.H.P.}}{V^3}$ at moderate speeds for the form never varies very rapidly in ordinate or character, and if the trial results show any marked variations, the data should be closely examined, and such things as depth of water, state of sea, excessive trim, etc., should be looked into.

If a model of the ship has been tested in an experiment tank, the analysis then proceeds as follows :—The curve of effective horse-power (E.H.P.) for the ship at the displacement as on trial, can be closely estimated from the model resistance experiments. If model screw experiments have been made the hull efficiency (h) and wake (w) at the different speeds are known. If no such experiments have been made, these coefficients must be assumed.

In this way a curve of

$$\frac{\text{E.H.P.}}{h \times V^3} = \frac{\text{T.H.P.}}{V^3}$$

can be plotted.

The curve derived in this way is for naked ship. An allowance of from 3 to 8 per cent. must be added for shaft webbing and keels, the former for single screw and the latter for quadruple screws, and from 1 to 4 per cent. for air resistance, according to the height and extent of hull above water.

Knowing the dimensions of the propellers, their revolutions and speed of advance through the water, the power (H) exerted by them, and their efficiencies at each speed can be worked out in the manner already described. If the vessel has triple or quadruple screws, the $\frac{H}{V^3}$ for each screw or pair of screws is worked out separately, using for the pitch of the screw (the face pitch) $\times$ 1·02, or for turbine screws 1·04. The calculated values of $\frac{H}{V^3}$ obtained on each shaft in this way are added together and plotted as a curve of total $\frac{H}{V^3}$.

The curves of $\frac{\text{T.H.P.}}{V^3}$ and $\frac{H}{V^3}$ derived from the model and from the screws, should be similar in shape, and differ in ordinate value only in so much as may be due to imperfect estimation of appendage resistance, or to extra resistance due to a foul bottom. Such differences in general ordinate value between the two curves can very often be adjusted by assuming a slightly different pitch for the propellers, or varying the wake factor. The true pitch of a propeller is never properly known, and the factors already given by which it can be derived from the face pitch, are empirical in character and vary slightly with shape of blade section and other makers' characteristics. Provided the hull efficiency is not greater than unity, which is generally the case, the $\frac{H}{V^3}$ derived

from the propellers should not be less than the $\frac{\text{T.H.P.}}{V^3}$ obtained from the model experiments.

Knowing the power H developed by each propeller and its efficiency, the power delivered to it (D.H.P.) can be calculated $\left(\text{being simply } \frac{H}{\text{screw efficiency}}\right)$, and can be plotted in the form of $\frac{\text{D.H.P.}}{V^3}$. The curve of $\frac{\text{D.H.P.}}{V^3}$ should differ in value from the $\frac{\text{measured shaft horse-power (S.H.P.)}}{V^3}$ by an amount due to the friction of the bearings (and thrust block, if the torsion meters are fixed forward of this). If indicated power has been measured, the difference between the $\frac{\text{I.H.P.}}{V^3}$ curve and the $\frac{\text{D.H.P.}}{V^3}$ curve is due to shaft and thrust block friction, main engine losses, and power taken up by auxiliary machinery worked off the main engines.

Since the friction, and to a large extent the engine losses, will vary with the load or torque on the shaft, if a curve of $\left(\frac{\text{I.H.P.}-\text{D.H.P.}}{\text{revolutions}}\right)$ be plotted to a base of $\left(\frac{\text{D.H.P.}}{\text{revolutions}}\right)$, it should give a fairly straight line. If the spots for the various trials do not come fair, some slight adjustment of the wake may help to smooth out discrepancies, but it must be remembered that any variation of D.H.P. effected in this way will affect the comparison of the T.H.P. obtained from the towing experiments, with the H values calculated from the screw particulars. This curve or line really shows the torque which is wasted in the engine, etc., to a base of torque delivered to the screw, and upon its general ordinate value depends the total efficiency of the engine and transmission, this efficiency being given by $\frac{\text{D.H.P.}}{\text{I.H.P.}}$

The efficiency of the engine obtained in this way must be regarded only as approximate, since it depends upon the wake of the ship and pitch of the propeller, and these are in most cases a

little indeterminate. If, however, every care has been taken in the trials, the possible error should not be more than about 3 per cent. The E.H.P. estimated from the model experiments is fairly accurate, and the screw efficiency is also fairly reliable. The uncertainty lies mainly between the ratio $\frac{\text{E.H.P.}}{H \text{ calculated from the screws}}$ and the efficiency of engines. The product of the two must be constant, and any variation of the wake which lowers the one must increase the other, the ratio $\frac{\text{E.H.P.}}{\text{I.H.P.}}$ remaining the same. In the case of vessels whose shaft horse-power has been measured there is not so much uncertainty, and the analysis is more satisfactory in this respect.

One of the important objects of such analysis is to see if cavitation is taking place. This shows itself in the first place by the attitude of the curves of $\frac{RP}{V}$ and $\frac{\text{I.H.P.}}{V^3}$ for ship. Both of these increase rapidly in value when cavitation commences. But a rapid increase in their value at high speeds is not in itself a sure indication of cavitation; it may be due to the ship resistance increasing at an abnormal rate.

This would be shown by the $\frac{\text{E.H.P.}}{V^3}$ curve, which should be either constant in ordinate value or increase gradually with speed. A sudden increase in value of $\frac{\text{E.H.P.}}{V^3}$ denotes that the form is for some reason unsuitable for the speed and the resistance abnormal. In this case the $\frac{R \times P}{V}$ and $\frac{\text{I.H.P.}}{V^3}$ will also increase rapidly, and other means than that indicated above must be adopted to detect the presence of cavitation. But if the $\frac{\text{E.H.P.}}{V^3}$ curve is fairly straight at those speeds at which the revolutions and power show a rapid increase, it is fairly certain that cavitation has developed.

A further check can be obtained from the values of (I.H.P.—D.H.P.). When cavitation is present the power estimated from

the revolutions exceeds that actually developed in the screws, and the estimated D.H.P. becomes too great and gives a fictitious, and correspondingly low value of (I.H.P.—D.H.P.) and an equally low value of $\frac{\text{E.H.P.}}{H}$.

Such an analysis as the above therefore gives a reliable indication of the worst evil a screw may meet, and also serves to give data as to both the engine and propeller efficiency, and enables these to be partially separated from each other. It also tells the designer the approximate ratio of driving face pitch to analysis pitch which must be used in making estimates from Froude's data for any particular make and style of propeller.

Tabulation and Plotting of Trial Results.—Without model experiments analysis of steam trial results cannot be carried very far, and what is usually done can only be regarded as tabulation of results for record and comparison. Some of the curves previously mentioned can, however, be plotted in order to check the results in themselves. For record purposes values of $\frac{\text{I.H.P.}}{\Delta^{\frac{2}{3}} \times V^3}$, or preferably $\frac{\text{I.H.P.}}{\begin{pmatrix}\text{wetted}\\ \text{surface}\end{pmatrix} \times (V^3)}$ should be tabulated, or plotted as ordinates to a base of $\frac{V}{\sqrt{P \times L}}$, P being the prismatic coefficient of the form and L the immersed length on which this coefficient has been calculated. This plotting takes account of two important factors in resistance, viz., the actual wetted surface, and the wave-making speed at which the ship may be running. If any humps occur in the plotted curve, they should fall at known values of $\frac{V}{\sqrt{P \times L}}$, viz., 1·34, ·95, ·76, etc. (see also Table 8).

Many factors influence the value of the ordinate, but the above is a ready and approximate method of comparing all types, and a fairly accurate one for comparing a number of forms of any given type. Table 34 gives particulars of a number of ships treated in this way.

Example of the Analysis of Steam Trial Results.

Displacement on trial . . .	8,500 tons.
Engines	triple expansion.
Propeller particulars :—	
Diameter	16·0 feet.
Driving face pitch . .	20·25 feet uniform.
Number of blades . . .	3
Developed blade area . .	70 square feet.
Clearance of blade tips and hull plating	13 inches.

Lines 2, 3, and 4 of the following table give the measured speed, indicated horse-power, and revolutions of engines.

The analysis pitch of the propeller $=20{\cdot}25\times1{\cdot}02=20{\cdot}67$ feet.

The analysis pitch ratio $=\dfrac{20{\cdot}67}{16{\cdot}0}=1{\cdot}29=p$.

The disc area ratio $=\dfrac{70+13}{\pi(8{\cdot}0)^2}={\cdot}414$.

The 13 square feet added in the last line is the allowance for the blade outline if produced to the shaft centre (see note (*d*) § 75).

Thrust factor from Fig. 36, *i.e.*, "B" . . . ·106
Hull efficiency assumed . . . . . ·98

$$\frac{p+21}{p}BD^2=\frac{22{\cdot}29}{1{\cdot}29}\times{\cdot}106\times(16{\cdot}0)^2=469 \text{ (see § 75).}$$

Thrust horse-power delivered by the screw (line 15 of the table) $=469\times Y\times(V_1)^3$.

The trials show a transmission efficiency of ·85 at service speeds, which is fairly good. The thrust block friction, which amounts to about 3 per cent. of the power transmitted, is taken into account in the above efficiency. The further analysis of this is dealt with on page 225.

Line 7 gives the power estimated from the tank experiments, and should agree with the figures in line 15, *i.e.*, the powers calculated from the known particulars of the propellers. The discrepancy may be due in part to rough water or to the screw not developing the calculated powers owing to too little clearance, which will also lower the efficiency. The latter assumption gives about the same D.H.P. value, but makes the "H" value lower. It may also be due to the reputed pitch being too high. A slightly lower pitch would improve line 21, but would reduce the engine efficiency shown by line 20.

If the figures in line 7 were divided by $\Delta^{\frac{2}{3}}V^3$ and plotted to a base of speed with the figures in line 9, the two curves should

TABLE 33.

Line	Item.		Runs.				
			1 & 2	3 & 4	5 & 6	7 & 8	9 & 10
2	Speed in knots	V	11·0	12·5	14·0	15·0	15·5
3	I.H.P., one shaft . . .	I	900	1,350	1,940	2,310	2,640
4	Revolutions in hundreds per minute	R	·60	·695	·795	·850	·89
5	½ (E.H.P.), from tank tests. .	E	400	625	925	1,175	1,325
6	E with 8 per cent. added for air and shaft tube resistance . .	—	432	675	1,000	1,270	1,432
7	Line 6 divided by hull efficiency (·98)	T.H.P.	441	689	1,020	1,294	1,461
8		$\frac{RP}{V}$	1,129	1·15	1·17	1·17	1·187
9	$\frac{I}{\Delta^{\frac{2}{3}} V^3}$	—	·00161	·00165	·00169	·00169	·00169
10	$\frac{\text{Line 7}}{(\text{velocity})^3}=$	$\frac{\text{T.H.P.}}{V^3}$	·332	·352	·372	·383	·393
11	Wake factor assumed . .	w	·17	·16	·16	·15	·14
12	$\left(\frac{V}{1+w}\right)$ speed of screw through wake water	V_1	9·40	10·78	12·07	13·04	13·6
13	$X=$.	$\frac{RP}{V_1}$	1·319	1·333	1·357	1·345	1·351
14	From Fig. 39	$Y=$	·00125	·00131	·00142	·00138	·0014
15	$\frac{p+21}{p} BD^2 V_1{}^3 Y=$	H	486	770	1,168	1,437	1,651
16	Efficiency of screw from Fig. 39, corresponding to above "X" values	—	·73	·73	·727	·73	·729
17	Correction for disc area from Fig. 37.	—	·002	·002	·002	·002	·002
18	Correct efficiency . . .	η	·732	·732	·729	·732	·731
19	Power on the shaft at the screw= $\frac{H}{\eta}=$	D.H.P.	664	1,052	1,602	1,960	2,259
20	Efficiency of engine, including thrust block and shaft friction.	$\frac{\text{D.H.P.}}{I}$	·74	·78	·825	·85	·855
21	Line 7 divided by line 15 . .	$\frac{\text{T.H.P.}}{H}$	·91	·90	·88	·91	·89
22	Propulsive coefficient (line 5 divided by line 3) . . .	—	·444	·463	·477	·51	·503

show about the same attitude to the base. The same remark applies to line 15, the attitude of which can be altered a little by adjusting the wake at the different speeds, so as to bring the results into better agreement.

This example, containing as it does some discrepancies, has been chosen in order to show how to deal with them. An intimate knowledge of the trials, or comparison of the trials, of sister ships is sometimes necessary for the clearing up of some of the doubtful points.

TABLE 34.

Measured Mile Results.

Ship.	Length (L) (feet).	Breadth (feet).	Draft (feet).	Dispt. (tons).	Wetted Surface (S) (sq. ft.).	Pris. Coefft. (P.).	Speed V (knots).	I.H.P.	$\frac{V}{\sqrt{PL}}$.	$\frac{\text{I.H.P.}}{SV^3}$.	No. of Screws.	Remarks.
Kirk's No. 1	342	38·0	19·3	4,500	19,350	·658	11·52	1,431	·77	·000048	1	
„ „ 2	342	38·0	19·0	4,415	19,140	·656	9·18	642	·61	·000043	1	
„ „ 3	344	39·0	18·0	4,235	18,900	·647	11·87	1,429	·80	·000045	1	4 ft. trim by stern.
„ „ 4	348	39·0	18·6	4,472	19,500	·653	12·94	2,106	·86	·000050	1	
„ „ 5	230	32·0	10·0	1,227	8,550	·625	9·32	528	·78	·000076	1	6 ft. trim by stern.
„ „ 6	230	32·0	14·8	2,034	10,850	·683	10·33	805	·82	·000067	1	
„ „ 7	240	32·0	12·6	1,693	10,216	·647	11·14	909	·89	·000064	1	
„ „ 8	285	35·0	14·7	2,710	13,950	·685	11·57	1,195	·83	·000055	1	
„ „ 9	190	25·5	12·2	1,115	7,300	·694	8·63	441	·75	·000094	1	
„ „ 11	204	26·5	10·5	885	6,700	·581	13·33	1,135	1·23	·000072	paddles	
„ „ 12	225	30·0	12·7	1,235	8,440	·533	12·66	1,450	1·15	·000084	2	
„ „ 13	98	18·0	5·0	133	1,940	·575	8·54	125	1·14	·000104	paddles	
„ „ 14	320	40·0	13·1	2,335	13,750	·522	13·89	2,252	1·07	·000061	1	8·6 ft. trim by stern.
Manning (brought to standard length).	400	68·2	26·2	9,670	32,800	·60	11·7	2,640	·76	·000050	1	·44 propulsive coefficients.
							16·05	6,850	1·04	·000051		·50
							20·4	17,550	1·32	·000063		·54
							23·3	30,900	1·51	·000075		·55
Mercantile vessel	400	51·4	26·0	11,600	36,000	·79	10·9	2,400	·61	·000052	1	
Dolphin	240	32·0	13·8	1,413	9,200	·61	15·5	2,144	1·28	·000063	1	
Mercantile vessel	400	62·5	24·2	10,900	32,000	·724	14·4	5,600	·85	·000059	1	
Passenger vessel.	315	33·5	14·8	2,480	13,500	·65	12·4	1,000	·87	·000039	1	
							15·0	1,950	1·05	·000043		
City of Rome	542	52·0	21·46	11,200	38,250	·71	18·2	13,690	·93	·000059	1	
City of Paris	523	63·4	21·2	11,400	37,900	·60	20·8	21,700	1·18	·000064	2	
Royal Sovereign (brought to standard length).	400	79·0	28·7	16,370	40,000	·672	19·2	19,650	1·17	·000069	2	
Iris	300	46·0	18·1	3,290	18,600	·54	12·28	1,833	·97	·000053	2	With final 4-blade propellers.
							16·56	5,108	1·30	·000060		
							18·57	7,714	1·46	·000065		
Lepanto	400	72·7	30·1	14,784	36,300	·65	13·7	6,230	·85	·000067	2	
							15·9	10,330	·98	·000071		
							18·4	16,150	1·14	·000072		

Devastation . .	285	62·3	—	—	—	·75	13·8	—	·94	·00009	2	
Yorktown . .	230	36	13·8	1,680	9,650	·59	14·8	2,320	1·27	·000074	2	
							16·6	3,580	1·42	·000081		
Columbia . .	418	58·4	22·5	7,375	28,700	·52	22·8	18,000	1·55	·000053	3	
Colorado . .	502	69·5	23·9	13,670	44,250	·60	15·5	7,100	·89	·000043	2	·49 propulsive coefficients.
							19·0	12,600	1·09	·000042		·55
							22·24	25,000	1·28	·000051		·58
City of Lowell .	320	48	12·8	2,546	14,000	·575	16·2	2,727	1·19	·000046	2	
				2,445	13,850	·57	19·3	4,347	1·43	·000044		
Yacht (brought to standard length).	400	50·5	23·0	5,800	24,500	·60	25·3	—	1·63	·000076	2	
*Utah** . .	510	88·2	27·8	21,268	51,000	·62	12·0	4,400	·68	·000050	4	·5 propulsive coefficients (see footnote).
							19·23	17,980	1·08	·000050		·54
							21·04	28,136	1·18	·000059		·54
*Lusitania** .	760	87·5	32·7	37,000	82,500	·616	15·8	13,400	·73	·000041	4	— propulsive coefficients.
							20·4	29,500	·94	·000042		·52
							23·7	51,300	1·10	·000047		·49
							25·6	76,000	1·18	·000055		·47
Cushing . .	138	14·2	4·87	105	1,900	·565	22·5	1,760	2·55	·000082	2	
Torpedo - boat (brought to standard length).	400	41·5	12·3	2,700	16,500	·66	17·6	—	1·08	·000065	2	·43 propulsive coefficients.
							28·7	—	1·76	·000063		·63
							36·7	—	2·25	·000068		·61
							43·1	—	2·65	·000066		·59
							47·9	—	2·94	·000062		·58
T.B.D. *McDougal**	300	30	9·0	1,010	8,450	·595	8·1	177	·61	·00004	2	
							15·65	1,290	1·17	·00004		
							19·36	2,687	1·45	·000044		
							24·25	6,488	1·81	·000054		
							27·38	11,187	2·05	·000064		
							31·5	17,741	2·36	·000067		
Normannia . .	500	57·4	22·2	10,600	35,500	·64	10·1	1,560	·56	·000043	2	
							14·5	4,300	·81	·000040		
							18·6	9,600	1·04	·000042		
							20·7	16,240	1·16	·000051		
Sunbeam . .	159	27·5	13·4	550	5,250	·55	10·13	360	1·08	·000066	1	

* These vessels have turbine-driven propellers, and the powers given are measured on the shaft and are not "indicated." The propulsive coefficients for these are the ratios $\frac{\text{E.H.P.}}{\text{S.H.P.}}$.

Table 35A.

Wake and Hull Efficiency Values (Single-screw Ships).

Ship.	Length (feet).	Breadth (feet).	Draft (feet).	Displacement (tons).	Prismatic Coefficient.	Speed (knots).	Wake Factor (w).	Hull Efficiency.	Screw Efficiency.	$\frac{\text{E.H.P.}}{\text{I.H.P.}}$
*Manning**	188	32·7	12·3	1,000	·60	10·0 15·0	·11 ·12	·9 ·9	·68 ·68	·49 ·54
Vessel A.	400	58·1	18·7	5,800	·60	19·0	·18	·98	·70	·50
Flavio Gioja	247	41·6	16·0	2,500	·62	15·0	·24			
Great Eastern	680	82·5	30·0	27,400	·63	—	·34			
Charles Quint	315	33·5	14·8	2,480	·65	15·0	·24			
Vessel B.	400	58	21·5	13,600	·68	14·5	·24	·97	·70	·56
Comus	218	—	—	2,300	·68	—	·36			
Warrior	380	58·0	26·3	9,200	·69	14·4	·33	—	—	·43
City of Rome	542	52·0	21·5	11,200	·71	18·2	·33	—	—	·50
Servia	500	52·0	25·2	12,350	·71	17·0	·34	—	—	·51
Monarch	330	57·5	23·7	8,100	·73	15·0	·37	1·0	—	·45

* Exhaustive trials of a 37·6 feet model of this vessel showed a considerable drop in hull efficiency if the distance from propeller to sternpost was much less than 6 inches. This was probably due to eddy-making, but the siding of the post is not known.

Note.—High prismatic coefficients in the after-body, although leading to high wakes, do not necessarily give high hull efficiency. If eddy-making occurs, the break up of the water will cause a drop in screw efficiency (see § 80).

TABLE 35B.

Wake and Hull Efficiency Values (Twin-screw Ships).

Ship.	Length (feet).	Breadth (feet).	Draft (feet).	Displacement (tons).	Prismatic Coefficient.	Speed (knots).	Wake Factor (w).	Hull Efficiency.	Screw Efficiency.	$\frac{\text{E.H.P.}^*}{\text{I.H.P.}}$
Mercury	300	46·0	22·0	3,730	·53	18·9	·06	—	·65	·536
Iris† :—										
Original 4-bladed screws						16·6			·52	·33
Original 2-bladed screws	300	46·0	18·1	3,290	·55	15·7	·06	·96	·62	·45
Final 4-bladed screws						18·6			·66	·50
Phaeton	300	46·0	20·5	4,300	·56	18·7	·09	—	·67	·59
Great Eastern	680	82·5	30·0	27,400	·63	—	·17			
"Admiral" class	325	68·0	25·2	8,900	·64	14·0	·12			
Inflexible (old)	320	75·0	23·9	11,100	—	14·7	·38	—	·64	·51
Italia	400	72·8	28·3	13,850	·65	—	·14	1·0	·68	·48
Collingwood	325	68·0	23·8	8,200	·66	16·8	—	—	·68	
Conqueror	270	58·0	24·0	6,200	·67	15·5	·16	—	·67	·56
Formidable	400	75·0	26·7	14,900	·68	18·0	·15	·97	—	See ‡ below.
Majestic	390	75·0	27·5	14,850	·69	17·0	·16	·94	—	
Devastation	285	62·3	27·5	9,300	·75	13·3	·18	—	·68	·46

* The E.H.P. used in calculating this ratio includes that for the shaft tubes and bossings.

† *Iris* propellers :—	Diameter (feet).	Mean Pitch (feet).	Total Developed Area (sq. feet).	Revs. at above Speeds.	Remarks.
Original 4-bladed screws	18·5	18·2	194	91	All screws have the same shaft centres.
Original 2-bladed screws	18·5	18·2	97	89	
Final 4-bladed screws	16·3	16·3	144	97	

‡ *Formidable* and *Majestic* propulsive coefficients $\left(\frac{\text{naked E.H.P.}}{\text{I.H.P.}}\right)$ may be taken as ·48 at speeds given.

TABLE 35C.

Wake and Hull Efficiency Values (Twin-screw Ships).

The length in every case is 400 feet.

Ship.		Breadth (feet).	Draft (feet).	Displacement (tons).	Prismatic Coefficient. Fore-body.	Prismatic Coefficient. After-body.	Wake Factor (w).	Hull Efficiency.	Remarks.
Torpedo-boat destroyers.	1	37·0	10·9	1,930	·57	·54	·04	1·0	Built by Thornycroft.
	2	40·0	10·6	1,920	·51	·54	−·01	·98	„ Lairds.
	3	37·3	11·4	2,010	·53	·58	·01	·98	„ Thornycroft.
	4	40·5	11·1	2,020	·54	·605	−·01	·97	„ Lairds.
	5	38·1	10·8	2,110	·60	·613	−·01	·97	„ Palmers.
Cruiser .	1	71·2	26·2	11,080	·56	·57	·10	·983	Numbers 1 and 2 are similar to 3, with finer after-body. Number 1 has the same clearance as 3, but 2 has slightly greater clearance.
	2				·56	·57	·08	·985	
	3				·58	·59	·10	·994	
	4	63·4	23·2	8,400	·55	·59	·06	·98	
	5	48·6	18·0	5,020	·57	·60	·05	·97	
	6	52·5	22·0	6,500	·53	·61	·07	·97	
Passenger vessel .		57·0	18·0	6,400	·59	·61	·04	·98	
Cruisers .	7	48·0	18·0	4,920	·53	·62	·05	·97	Relative rotative efficiency out-turning ·985, in-turning 1·0.
	8	63·7	22·5	8,550	·567	·62	·08	·97	
	9	56·0	21·7	7,160	·568	·62	·08	·97	
Shallow draft vessel		72·2	16·6	7,120	·58	·62	·16	1·02	Has Thornycroft stern.
Mercantile vessel .		50·0	19·5	7,200	·68	·65	·15	·95	
Cruiser or liner A .		52·0	18·5	6,500	·59	·67	·09	·98	
Battleship A .		76·0	26·6	13,900	·62	·68	·10	·99	
Cruiser or liner B .		52·0	18·5	6,500	·61	·683	·09	·98	
Battleship B .		76·0	26·7	13,800	·60	·684	·10	·99	
Cruiser or liner C .		51·3	18·5	6,500	·64	·70	·09	·95	Relative rotative efficiency = 1·02.
Luke's model .		58·8	19·6	7,650	·63	·72	·20	1·02	
Battleship C .		77·0	28·2	16,000	·64	·74	·17	·96	
Mercantile vessel .		54·0	19·3	8,400	·72	·74	·20	·99	

NOTE.—The relative rotative efficiency is within 1 per cent. of unity unless special values are given.

TABLE 35D.

Wake and Hull Efficiency Values (Twin-screw Ships).

The length in every case is 400 feet.

Ship.	Breadth (feet).	Draft (feet).	Displacement (tons).	Block Coefficient.	Speed (knots).	Wake Factor (w).	Hull Efficiency (h).	Relative Rotative Efficiency.	Diameter of screws (feet).
Cruiser 1	42·4	15·5	3,790	·50	25·0	·03	·96	·98	16·7
Battleship 1	73·5	24·7	10,540	·51	21·0	·11	·98	·99	17·2
Cruiser 2	71·3	27·1	11,450	·52	21·0	·09	·97	·99	17·5
Battleship 2	67·5	23·6	9,540	·524	21·0	·08	·94	·93	17·2
						·07	·96	·95	17·2
						·09	·95	·96	17·2
						·11	·98	·99	16·6
Cruiser 3	68·8	23·3	9,820	·535	21·0	·04	·98	·99	18·8
						·08	1·0	·99	18·8
Merchant vessel	70·1	27·0	12,860	·60	15·2	·20	1·04	1·01	17·0

Table 36.

Correction of (C) Value, passing from a Ship Length of 400 Feet to any other Length.

Length of Ship (feet).	Addition to (C) Value.	Length of Ship (feet).	Deduction from (C) Value.
100	·09	400	
150	·066	450	·007
200	·045	500	·013
250	·03	600	·024
300	·013	700	·033
350	·009	800	·041
400	—	900	·048
—	—	1,000	·054

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SPECIAL INDEX

This index gives the numbers of the sections in which reference is made to vessels of definite prismatic coefficient.

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